ADAPTIVE $L_2$ DISTURBANCE ATTENUATION OF HAMILTONIAN SYSTEMS WITH PARAMETRIC PERTURBATION AND APPLICATION TO POWER SYSTEMS

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ABSTRACT

This paper deals with the problem of $L_2$ disturbance attenuation for Hamiltonian systems. We first show that the $L_2$ gain from the disturbance to a penalty signal may be reduced to any given level if the penalty signal is defined properly. Then, an adaptive version of the controller will be presented to compensate the parameter perturbation. When the perturbed parameters satisfy a suitable matching condition, it is easy to introduce the adaptive mechanism to the controller. Another contribution of this paper is to apply the proposed method to the excitation control problem for power systems. An adaptive $L_2$ controller for the power system is designed using the proposed method and a simulation result with the proposed controller is given.

KeyWords: Hamiltonian systems, $L_2$ disturbance attenuation, adaptive control, power system.

I. INTRODUCTION

In the last decade, there has been renewal of interest in developing systematic design methodologies for control of nonlinear systems. A powerful design technique for stabilization of nonlinear systems is passivity-based control (PBC) [1]. In the PBC framework, the controller design proceeds along two stages. The first stage is to render passive a map with a suitably defined storage function, and the second stage is to perform an output feedback with detectability condition. Moreover, in order to enhance the stability and robustness, $L_2V$ control law with the storage function $V(x)$ can be employed in the second stage [4,5]. For mechanical systems the design process has a physical meaning, e.g. the first stage can be carried out by shaping the potential energy of the system in such a way that the new potential energy function has a strict local minimum at the desired equilibrium, and the second stage is nothing but damping injection. As shown in [2] the procedure can be applied also to a large class of electrical and electromechanical systems described by Euler-Lagrange equations of motion.

Recently, the design technology has been extended to a broader class of systems described by port-controlled Hamiltonian (PCH) models. Indeed, the Hamiltonian function in PCH systems is the total energy, potential and kinetic energy in physical systems, and can play the role of Lyapunov function for the system. However, when the system is forced by external input such as set-point regulation, the Hamiltonian function dose not necessarily have a minimum at the desired operating point. In this case, we could employ a pre-feedback and shape the Hamiltonian function such that the closed loop system has Hamiltonian structure with the modified function to ensure stability. Furthermore, under some detectability conditions, asymptotic stability is also ensured. The problem of finding such a feedback law has been studied by [7,8], and the passivity-based control for...
PCH systems has been investigated by [3-10]. Also, several application examples have been illustrated by [11, 13].

In practical engineering, disturbance attenuation and parametric uncertainty are also important issues. PBC design method has been extended by many researchers to achieve γ-dissipativity [14] that not only guarantees asymptotic stability but also renders the $L_2$-gain from disturbance to a penalty signal less than a given level $\gamma > 0$. As PBC design, a key to solve the disturbance attenuation problem along this line is to construct a proper storage function that ensures the γ-dissipativity. Several effective methods have been reported by [15-18]. For nonlinear systems with unknown parameters, the PBC design methodology has been extended to include adaptive mechanism [6]. For example, if $L_eV$ involves unknown parameters in linear form, which caused by perturbation in the vector field $g(x)$ and the Lyapunov function $V(x)$, then a parameter adaptation law can be introduced into the $L_eV$ controller to compensation of the unknown parameters [20].

In this paper, we are interested in the problem of $L_2$ disturbance attenuation for the Hamiltonian systems with parametric perturbations. It will be shown that for a given $\gamma > 0$, γ-dissipativity can be achieved by making a sufficiently large damping injection in the second design stage only, if the penalty signal is properly defined. Then, we will consider the case when the Hamiltonian system involves parametric perturbations. We shall show that if the perturbations satisfy a suitable matching condition, then an adaptation mechanism can be added to the feedback controller to estimate the controller parameters corresponding to the parameter perturbations. It is interesting to note that the unknown parameters may enter nonlinearly in the control law and/or the energy function, and only an assumption of linear reparametrization is needed. Essentially, the proposed controller is in $L_eV$ form, where the Hamiltonian function will plays as the role of Lyapunov function $V(x)$. However, when the system involves parameter perturbation, the nominal part of the Hamiltonian function can serve as storage function, and the adaptive mechanism is introduced to cover not only the perturbation in $L_eV$ term but also the damping and the structure matrix related term. Hence, the proposed controller is based on more general setting.

Another aim of this paper is to apply the proposed approach to power systems. Application examples of the PBC methodology on the power system have been shown by [19,20]. An adaptive $L_2$-disturbance attenuation control has been illustrated by [21], and an Hamiltonian system view has been reported by [13]. In this paper, it will be shown that the proposed approach can be applied to the excitation control problem of the power system with electrical parameter perturbations. Finally, simulation results on a power engineering professional system will be given.

II. PRELIMINARIES

Consider the port-controlled Hamiltonian system with dissipation described by

$$
\begin{align*}
\dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\
y &= g^T(x) \frac{\partial H}{\partial y}(x)
\end{align*}
$$

(1)

where $x \in \mathcal{X}$, an $n$-dimensional manifold, $u, y \in \mathbb{R}^n$ are the input and output, respectively; $H : \mathbb{R}^n \rightarrow \mathbb{R}$ represents the total stored energy called Hamiltonian function, $R(x)$ is a skew-symmetric structure matrix, and $R(x)$ is a non-negative symmetric matrix. The notation $\frac{\partial H}{\partial x}$ is defined by $\frac{\partial H}{\partial x} = \left[ \frac{\partial H}{\partial x_1}, \ldots, \frac{\partial H}{\partial x_n} \right]^T$.

An interesting property of this class of the systems is that the Hamiltonian function $H(x)$ can play the role of Lyapunov function for stability analysis, i.e. if $H(x)$ admits a strict minimum at $x_0$, then $x_0$ is a stable equilibrium of the unforced systems

$$
\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x)
$$

(2)

Usually $x_0$ corresponds to the zero operating point, therefore the control input can be used to shape the Hamiltonian function $H(x) \rightarrow H_0(x)$ such that under a proper feedback $u = \alpha(x)$, the closed loop system has the Hamiltonian structure, i.e.

$$
[J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)\alpha(x) = [J(x) - R_0(x)] \frac{\partial H_0}{\partial x}(x)
$$

(3)

and $H_0(x)$ admits a strict minimum at a desired equilibrium $x_0$. This problem has been investigated by [3] and [7].

The main objective of this paper is the disturbance attenuation. We will start with the model (1) to which we add a disturbance

$$
\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u + w
$$

(4)

where $w \in \mathbb{R}^n$ is an essentially bounded unknown disturbance such that the trajectory $x(t)$ remain in $\mathcal{X}$ for any initial state $x(0) \in \mathcal{X}$. 
The $L_2$ disturbance attenuation objective is as follows: Given a desired equilibrium $x_0 \in \mathbb{R}^n$, a penalty signal $z = q(x)$ ($q(x_0) = 0$) and a disturbance attenuation level $\gamma > 0$. Find a feedback control law $u = k(x)$ and a positive definite (with respect to $x_0$) storage function $V(x)$ such that for the closed loop system consisting of (4) with the feedback law, the $\gamma$-dissipation inequality holds along all trajectories remained in $\mathcal{X}$, where $Q(x)$ ($Q(x_0) = 0$) is a given non-negative definite function.

As pointed out by [22], the $\gamma$-dissipation inequality (5) ensures the following performance.

[S1] $L_2$-gain from the disturbance $w$ to the penalty signal $z$ is less than the given level $\gamma$.

[S2] When $w \equiv 0$, $x_0$ is a Lyapunov stable equilibrium of the unperturbed system in $\mathcal{X}$. Furthermore, it will be asymptotically stable if the following implication is true

\[ Q(x) + \frac{1}{2} \|q(x)\|^2 = 0 \Rightarrow x = x_0 \]

[S3] If $w$ is square integrable then $x$ is uniformly bounded.\(^1\)

### III CONTROL LAW DESIGN

In this section we present our controller design, treating first the case without parameter uncertainty and then adding adaptation.

#### 3.1 Known parameter case

Suppose that for the system (4) there exists such a feedback $u = \alpha(x)$ that preserves Hamiltonian structure with a modified Hamiltonian function $H_c(x)$ and a symmetric non-negative matrix $R_c(x)$, i.e. (3) holds, and $H_c$ admits a strict minimum at the desired equilibrium $x_0$. A method for seeking such a feedback law $\alpha(x)$ has been proposed by [3] and [7].

Let the penalty signal be defined as follows

\[ z = h(x)g^\top(x) \frac{\partial H_c}{\partial x}(x) \tag{7} \]

where $g^\top \frac{\partial H_c}{\partial x}$ is a standard output of Hamiltonian system and $h(x)$ ($h(x_0) = 0$) is a weighting matrix.

\(^1\)To ensure boundedness of the state for bounded, but not square integrable $w$, additional conditions on $Q(x) + \frac{1}{2} \|q(x)\|^2$ should be imposed, e.g., to enforce exponential stability.

First, we will show that in this case, the $L_2$ disturbance attenuation objective can be achieved by damping injection, i.e. the Hamiltonian function $H_c$ can serve as the storage function for the closed loop system if we insert a proper feedback.

**Theorem 1.** Consider the system (4) with the penalty signal (7). For any given $\gamma > 0$, the $L_2$ disturbance attenuation objective is achieved by the following feedback control law

\[
\begin{align*}
\dot{u} &= \alpha(x) + \beta(x) \\
\dot{\beta}(x) &= -\frac{1}{2} \left\{ \gamma^2 I + h^\top(x)h(x) \right\} g^\top(x) \frac{\partial H_c}{\partial x}(x)
\end{align*}
\tag{8}
\]

**Proof.** Note that, under the feedback (8), the closed loop system with the modified Hamiltonian function $H_c$ can be represented by

\[
\dot{x} = \left[J(x) - R_c(x)\frac{\partial H_c}{\partial x}(x) + g(x)(\beta(x) + w)\right]
\]

\[ z = h(x)g^\top(x) \frac{\partial H_c}{\partial x}(x) \tag{9} \]

Along any trajectory of this system, a straightforward calculation gets

\[
\begin{align*}
\dot{H}_c &= -\frac{\partial H_c}{\partial x}(x)R_c(x)\frac{\partial H_c}{\partial x}(x) \\
&\quad + \frac{\partial}{\partial x}(x)g(x)(\beta(x) + w) \\
&= -\frac{\partial H_c}{\partial x}(x)R_c(x)\frac{\partial H_c}{\partial x}(x) + \frac{\partial^2 H_c}{\partial x^2}(x)g(x) \\
&\quad \times \left[ \beta(x) + \frac{1}{2} \left\{ \gamma^2 I + h^\top(x)h(x) \right\} g^\top(x) \frac{\partial H_c}{\partial x}(x) \right]
\end{align*}
\tag{10}
\]

Hence, by substituting $\beta(x)$ into the right side and setting the non-negative definite function

\[ Q(x) = \frac{\partial^2 H_c}{\partial x^2}(x)R_c(x)\frac{\partial H_c}{\partial x}(x) \]

we have

\[
\dot{H}_c + Q(x) \leq \frac{1}{2} \left\{ \gamma^2 \|w\|^2 - \|\beta\|^2 \right\}, \quad \forall w \tag{11}
\]

This means the Hamiltonian function $H_c$ serves as the storage function for the closed loop system.
Remark 1. The PBC design mentioned in Section 1 allows us to calculate the closed loop system that shapes the energy and adds damping, i.e., \( H(x) \to H_s(x) \) and \( R(x) \to \hat{R}(x) \) such that the unforced system when \( w = 0 \) is stable at the desired equilibrium \( x_0 \). Theorem 1 shows that in order to furthermore render the closed loop system dissipative, we only need to inject additional damping in the second stage

\[
\dot{R}_s(x) \to R_s(x) + g(x) \frac{1}{2} \left[ \frac{1}{T^2} \right] + h(x) \dot{h}(x) g^T(x)
\]

which provided by the additional feedback \( \dot{h}(x) \). It should be noted that \( \dot{h}(x) \) has \( L_\infty \) structure, i.e., it is of the form \( \dot{h}(x) \), where \( \dot{h}(x) \). The relation between the disturbance attenuation level \( \gamma \) and the gain \( \dot{h}(x) \) is given clearly in Theorem 1.

Remark 2. As mentioned in Section 2, the closed loop system with the controller (8) ensures the performance [S1], [S3] and stability of the equilibrium. In order to achieve asymptotic stability, the weighting matrix \( h(x) \), the structure matrix \( \dot{h}(x) \) and Hamiltonian function \( H(x) \) should be chosen such that (6) is satisfied, i.e.

\[
\tilde{H} \in \frac{\partial H}{\partial x} \left\{ R(x) + \frac{1}{2} \left[ \frac{1}{2} \right] h(x) \dot{h}(x) g^T(x) \right\} \partial H = 0 \implies x = x_0 \quad (12)
\]

Given that \( H(x) \) has an isolated minimum at \( x_0 \), this technical assumption is locally satisfied if the term inside the brackets is full rank. This condition depends on the degrees of freedom available for damping injection and the disturbance attenuation performance, hence essentially determined by the rank of \( g(x) \) and \( \dot{h}(x) \).

3.2 Adaptive case

We now consider the case when the model of system (4) involves parameter perturbations. Let the parameter perturbations be represented by a constant vector \( p \) whose nominal value is zero. Suppose that the system (4) is represented by

\[
\dot{x} = [J(x, p) - \hat{R}(x, p)] \frac{\partial H}{\partial x}(x, p) + g(x) (u + w) (13)
\]

For simplicity, we denote \( J(x, 0) = \hat{J}(x) \), \( \hat{R}(x, 0) = \hat{R}(x) \) and \( H(x, 0) = H(x) \).

In this case, the modified Hamiltonian function \( \hat{H}_s \), \( \hat{J}_s \), \( \hat{R}_s \) and the pre-feedback law \( \alpha \) will involve the perturbed parameter vector \( p \), i.e. under the state feedback

\[
u = \alpha(x, p) + \nu
\]

the closed loop system can be represented as follows

\[
\dot{x} = [J(x, p) - \hat{R}(x, p)] \frac{\partial H}{\partial x}(x, p) + g(x)(u + w)
\]

Decompose all functions related to the perturbed parameters \( p \) as follows:

\[
\frac{\partial H}{\partial x}(x, p) = \frac{\partial H}{\partial x}(x, \alpha) + \Delta \hat{H}(x, p),
\]

\[
\dot{R}_s(x) = \hat{R}(x) + \Delta \hat{R}(x, p),
\]

\[
J_s(x, p) = \hat{J}_s(x) + \Delta \hat{J}_s(x, p),
\]

\[
\alpha(x, p) = \alpha(x) + \Delta \alpha(x, p)
\]

where \( \Delta \hat{H}(x, 0) = 0 \) and \( \Delta \hat{R}(x, 0) = 0 \) and \( \Delta \hat{J}_s(x, 0) = 0 \) for simplicity.

Since the parameter perturbation vector \( p \) is unknown, we substitute the nominal function \( \alpha(x) \) with the nominal parameter for the previous feedback \( \alpha(x, p) \), and design an adaptive controller with parameter estimation

\[
\hat{\theta} = \text{Proj} \{ \theta \}
\]

where \( \hat{\theta} \) is an estimate vector and Proj is the well-known projection operator associated to the convex set \( \bar{\theta} = \{ \theta \in R^q \mid P(\theta) \leq 0 \} \). For some convex function \( P(\theta) \), see e.g., Appendix E of [6]. As usual, the projection is introduced to ensure the parameters remain bounded.

Therefore, our goal is to seek the functions \( \beta(x, \hat{\theta}) \), \( \phi(x) \) and to modify the energy function \( \hat{H}(x) \) in such a way that the \( \gamma \)-dissipation inequality

\[
\dot{U} + Q(x) \leq \frac{1}{2} \left[ \frac{1}{2} \right] - \frac{1}{2} [\hat{\theta}]^T \hat{\theta}
\]

holds for a properly constructed storage function \( U(x, \theta) \). The problem of seeking such a desired \( \beta(x, \hat{\theta}) \) and \( \phi(x) \) is referred to as adaptive \( L_\infty \) disturbance attenuation problem. A solution of this problem will be given in the following.

Theorem 2. Consider the system (13) with the penalty signal (7). Assume that there exists a function \( \Psi(x) \) such that

\[
[J_s(x, p) - \hat{R}_s(x, p)] \Delta \hat{J}_s(x, p) - g(x) \Delta \hat{R}_s(x, p)
\]

\[
geq g(x) \Psi^T(x) \chi \theta
\]

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hold for all \(x\). Then, for any given \(\gamma > 0\), the adaptive \(L_2\) disturbance attenuation problem is solved by

\[
\dot{\theta}(x, \dot{\theta}) = -\frac{1}{\gamma} \left[ I + \kappa^T \dot{\theta}(x) \right] g^T(x) \frac{\partial H}{\partial x}(x) - \Psi^T(x, \dot{\theta})
\]

\[
\dot{\phi}(x) = \Gamma \Psi(x) g^T(x) \frac{\partial H}{\partial x}(x)
\]

(19)

where \(\theta \in \mathbb{R}^q\) denotes a constant parameter vector, \(\dot{\theta}\) is an estimation of \(\theta\) and \(\Gamma = \text{diag}\{\rho_1, \rho_2, \ldots, \rho_q\}\), \(\rho_i > 0\) \((i = 1, 2, \ldots, q)\), is the parameter adaptation gain matrix.

Proof. For the system (13), we perform the feedback (16) with the functions given by (19). Then, using the condition (18), the closed loop system can be presented by

\[
\dot{x} = [J(x, p) - R(x, p)] \frac{\partial H}{\partial x}(x) + g(x) \left(-\alpha(x) + \beta(x, \dot{x}) + w\right)
\]

\[
= [J(x, p) - R(x, p)] \frac{\partial H}{\partial x}(x) + g(x) (\Psi^T(x) \dot{\theta} + \beta(x, \dot{x}) + w)
\]

(20)

For this system, we modify the energy function \(H\) to generate the desired storage function \(U\) as follows:

\[
U(x, \dot{\theta}) = H(x) + \frac{1}{2} (\theta - \hat{\theta}(t))^T \Gamma^{-1} (\theta - \hat{\theta}(t))
\]

(21)

Then, along any trajectories of the system, we have

\[
\dot{U} = \dot{\theta}^T \Gamma^{-1} (\theta - \hat{\theta})
\]

\[
= \frac{\partial^2 H}{\partial \theta \partial x}(x) R(x, p) \frac{\partial H}{\partial x}(x) + \frac{\partial^2 H}{\partial \dot{\theta}\partial x}(x) g(x)
\]

\[
\times \left[ \frac{1}{2} \frac{1}{\gamma^2} I + \kappa^T \dot{\theta}(x) \right] g^T(x) \frac{\partial H}{\partial x}(x) + w
\]

\[
+ \dot{\theta}^T \Psi^T(x, \dot{\theta}) \dot{\theta} - \dot{\theta} \Gamma^{-1} (\theta - \hat{\theta})
\]

(22)

Note that, when \(\hat{\theta}\) is inside the set \(\Pi\), the parameter adaptation law is given by

\[
\dot{\hat{\theta}} = \phi(x) = \Gamma \Psi(x) g^T(x) \frac{\partial H}{\partial x}(x)
\]

(23)

Thus, by the same techniques used in the proof of Theorem 1, we obtained

\[
\dot{U} + Q(x) \leq \frac{1}{2} \frac{1}{\gamma^2} \| \dot{\theta} \| - H^T(\theta - \hat{\theta})
\]

(24)

The proof is completed invoking the properties of the projection operator, e.g., Lemma E.1 in [6].

Remark 3. It is interesting to compare our controller with the dynamic extension with adaptation for \(L_2-V\) control of [20]. In the latter paper, the \(L_2-V\) function is parameterized as unknown parameters which caused by unknown equilibrium, and an adaptation law is introduced to ensure the stability under the assumption \(L_2-V < 0\), i.e. the free system is stable even though the vector field \(f(x)\) and the Lyapunov function \(V(x)\) involve unknown parameters. As shown in the proof of Theorem 2, the nominal part of the Hamiltonian function \(H(x)\), not \(H(x, p)\), can serve as the role of Lyapunov function which ensures the stability by utilizing the Hamiltonian structure \(J(x, p)\) and \(R(x, p)\), and the adaptive mechanism is introduce to cover the perturbation in \(L_2-H(x, p)\) and \(H(x, p)\). We should underscore that the stabilization mechanisms in both works are different.

Remark 4. The Hamiltonian structure is utilized here-together with the matching assumption (18) on \(\theta\) to preserve a nominal part of the perturbed dynamics which is stable with the Lyapunov function corresponding to the unperturbed parameters, i.e. the first right hand term of (20), whose Lyapunov function is \(H(x)\), instead of \(H(x, p)\). This factorization is essential to be consistent with the (aforementioned) objective of robustifying the “nominal” equilibrium, and does not seem to be easy to carry out without the Hamiltonian formulation, that allows to use the particular properties of the matrices \(J(x, p)\) and \(R(x, p)\).

IV. APPLICATION TO POWER SYSTEMS

We will apply the proposed controller to the problem of excitation of power systems. The problem has been addressed by many researchers. A nonlinear control approach based on the exact linearization method has been proposed by [23], while PBC have been investigated by [12,13,19,20]. \(L_2\) disturbance attenuation problem has been also studied by [21,24] for the power system. The design proposed in [24] is based on \(\gamma\)-dissipativity, however, the physical energy was not taken account in constructing the storage function. In consequence, the controller has a rather complex structure. It should be noted that putting physics into the \(\gamma\)-dissipativity design will make the controller structure very simple, however, the penalty signal should be restricted to
the standard form as shown in Section 3.

In this Section, we will show that the power system forced by a constant excitation signal which is for setpoint regulation has the Hamiltonian structure. Therefore, applying the proposed design approach, the dissipativity can be achieved by injection additional damping only. Moreover, we will show that the controller can be easily extended to the adaptive version, if the generator electrical dynamics involves parametric uncertainty.

4.1 System description

A simplified single-machine infinite bus power system with silicon-controlled rectifier (SRC) direct excitor is as shown in Fig. 1. A model for excitation control of this system can be written as follows: [23]

Mechanical equations:

\[ \dot{\delta} = \omega(t) - \omega_0 \]  
\[ \omega = \frac{D}{M} (\alpha(t) - \omega_0) + \frac{\omega_0}{M} [P_n - P_e(t)] \]  

Generator electrical dynamics:

\[ E_{q}' = \frac{1}{T_d} E_{q}'(t) + \frac{1}{T_d} x_{ss} - x_{sc}' V_c \cos(\delta(t)) \]
\[ + \frac{1}{T_d} V_c(t) + w \]  

where \( P_e(t) = \frac{E_{q}'(t) V_c}{x_{ss}} \sin \delta \) is the active electrical power, \( \delta(t) \) and \( \omega(t) \) are angle and speed of the rotor, respectively. \( E_{q}'(t) \) is the transient EMF in the quadrature axis of the generator, \( V_c \) denotes the control input of the SCR amplifier of the generator, and \( w \) denotes the unknown disturbance which caused by faults in the line or loads level variation, etc.²

We only consider the excitation control loop. Hence, we assume that the mechanical input power \( P_m \) and the speed of synchronous machine \( \omega_0 \) are constants. Define the state variable by

\[ x_1 = \delta, \ x_2 = \omega - \omega_0, \ x_3 = E_{q}' \]  

Then, dynamics of the system can be represented by the following state space model:

\[ \begin{aligned}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= -D \omega x_2 - b_1 x_1 \sin x_1 + P \\
    \dot{x}_3 &= c \cos x_1 - c x_1 + V + w
\end{aligned} \]  

where the control input \( V(t) = \frac{1}{T_d} V_c(t) \), and the parameters are defined by

\[ D = \frac{D}{M}, \ b_1 = \frac{\omega_0 V_c}{x_{ss}}, \ c = \frac{\omega_0}{M} P_n, \]
\[ c = \frac{x_f v}{T_d}, \ c_l = \frac{1}{T_d} \]

As it is well-known (see e.g. [20] for a recent reference) if we insert a constant excitation input \( V(t) = 0 \), then the system with \( w = 0 \) has a local equilibrium \((x_{1e}, 0, x_{3e})\) satisfying

\[ \begin{aligned}
    b_1 x_{1e} \sin x_{1e} &= P \\
    c_l x_{2e} - c_l \cos x_{2e} &= \pi
\end{aligned} \]  

Actually, an energy-like function

\[ H(x) = \frac{1}{2} x_2^2 + b_1 x_1 (\cos x_2 - \cos x_1) \]
\[ - P(x_1 - x_{2e}) + \frac{1}{2} c_l \left(x_1 - x_{2e}\right)^2 \]  

qualifies as a Lyapunov function for the forced system (29) with the constant input

\[ V(t) = \frac{1}{T_d} V_c(t) \]  

where \( x_{1e} \) and \( x_{3e} \) is a locally unique solution of (30).

It is easy to check that \( H(x) \) admits a local strict minimum at \((x_{1e}, 0, x_{3e})\) and the forced system (29) with the feedback control

\[ V(t) = \frac{1}{T_d} V_c(t) \]  

can be represented by the following formulation with the Hamiltonian function (31):
\[
\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)(v + w) \tag{34}
\]

where \(g(x) = [0 \ 0 \ 1]^T\) and the structure matrices defined by

\[
J(x) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_{ru} & 0 \\ 0 & 0 & \frac{c_2}{b_2} \end{bmatrix}
\]

4.2 Control law design

Consider the penalty signal defined by (7) with the weighting function

\[
h(t) = \begin{bmatrix} q_1(x_1 - x_{ue}) \\ q_2(x_2 - x_{ue}) \\ q_3(x_3 - x_{ue}) \end{bmatrix}, \quad q_i \geq 0 (i = 1, 2, 3) \tag{36}
\]

Note that, the standard output is given by

\[
g^T(x)H(x) = \begin{bmatrix} b_1(c_x^1 - \cos x_1) + \frac{b_2c_r}{c_1}(x_2 - x_{ue}) \end{bmatrix}\tag{37}
\]

Hence, we have

\[
\|z\|^2 \leq \begin{bmatrix} b_1(c_x^1 - \cos x_1) + \frac{b_2c_r}{c_1}(x_2 - x_{ue}) \end{bmatrix}^2 + \sum q_i(x_i - x_{ue})^2 \tag{38}
\]

This implies that \(z(x) = 0\) if and only if \((x_1, x_2, x_3) = (x_{ue}, 0, x_{ue})\) or

\[
b_2(c_x^2 - \cos x_2) + \frac{b_4c_r}{c_1}(x_3 - x_{ue}) = 0
\]

When \(w(t)\) vanishes, this equality means \(x_3 = 0\) and from the locally uniqueness of (30), \(x_1 = x_{ue}\) and \(x_3 = x_{ue}\). By the equation (29), \(x_2 = -D_{ru}x_3\). This leads, from the positivity of parameter \(D_{ru}\), \(x_2 \to 0\). Therefore, \(\|z\|^2\) represents the perturbation of the state from the given equilibrium. The aim of control is rejection of the state perturbation under the disturbance and the uncertainty.

Our goal is as follows: For given desired equilibrium \((x_{ue}, 0, x_{ue})\) and any given disturbance attenuation level \(\gamma > 0\), find a feedback control law such that the unforced closed loop system has a stable equilibrium in \(x_e\), and has the property of \(\gamma\)-dissipativity described in Section 2.

As mentioned above, the system (29) forced by the constant input (32), \(\alpha(x) = \bar{u}\), has the Hamiltonian structure and the total energy function \(H\) admit a strict minimum at the desired equilibrium. Thus, applying Theorem 1 to the system (29) gets \(s\) desired feedback law as follows

\[
u = \frac{1}{\gamma^2} \left[ \frac{1}{2} h(x) + g^T(x)H(x) \right] = \frac{1}{\gamma^2} \left[ \begin{array}{c} c_1x_1 - c_1 \cos x_1 - \frac{1}{2} \sum q_i(x_i - x_{ue})^2 \\ \frac{b_2(c_x^1 - \cos x_1) + \frac{b_4c_r}{c_1}(x_3 - x_{ue})} \end{array} \right] \tag{39}
\]

where

\[
\frac{\partial H}{\partial x}(x) = \begin{bmatrix} b_1(x_1 - x_{ue}) - b_1c_r \cos x_1 + \frac{b_2c_r}{c_1}(x_3 - x_{ue}) \end{bmatrix} \tag{40}
\]

By the definition of \(R, H\), and \(h(x)\), we have

\[
\frac{\partial H}{\partial x}(x) = \begin{bmatrix} b_1(x_1 - x_{ue}) - b_1c_r \cos x_1 + \frac{b_2c_r}{c_1}(x_3 - x_{ue}) \end{bmatrix} \tag{41}
\]

i.e. the condition (12) in Remark 2 is obviously satisfied. Hence, the closed loop system with this controller achieves the performance [S1]-[S3] mentioned in Section 2.

We now design the adaptive controller for the power system. When a fault occurs or the structure of the network changed, the parameters of the electrical equation will change drastically, and so, the equilibrium \(x_{ue}\), \(x_{ue}\) will be changed. Therefore, it is reasonable in the practical power systems to consider the uncertainties in the coefficients of electrical equation \(c_L, c_T, x_{ue}\) and \(x_{ue}\).

We consider the parameter perturbation as follows

\[
c_1 \to c_1 + p_1, \quad c_T \to c_T + p_2, \quad \cos x_{ue} \to \cos x_{ue} + p_3, \quad x_{ue} \to x_{ue} + p_4
\tag{42}
\]

where \(p_1, p_2, p_3, p_4\) are bounded unknown constants.
Then, it is easy to check that the Hamiltonian function \( H(x, p) \), the structure matrices \( J_f(x, p), R_f(x, p) \) and the feedback \( \alpha(x, p) \) can be decomposed by
\[
\frac{\partial H_f}{\partial x}(x, p) = \frac{\partial H_f}{\partial x}(x) + \Delta_x(x, p),
\]
\[
R_f(x, p) = R_f(x) + \Delta_x(x, p),
\]
\[
J_f(x, p) = J_f(x) + \Delta_x(x, p),
\]
\[
\alpha(x) = \alpha(x) + \Delta_\alpha(x, p)
\]
with the perturbed functions defined by
\[
\Delta_x(x, p) = [0 \ 0 \ b_2 \theta'_2 + b_3 \theta'_3 (x_1 - x_{sn})]^T,
\]
\[
\Delta_\alpha(x, p) = \alpha(x) + \Delta_\alpha(x, p)
\]
where
\[
\theta'_2 = p_2 - \frac{c_2 + p_1}{c_2 + p_1} p_4, \quad \theta'_3 = \frac{c_2 + p_1}{c_2 + p_1} c_4,
\]
\[
\theta'_3 = \{ (c_7 + p_7)(x_{sn} + p_4) - (c_7 + p_7) \cos(x_{sn} + p_4) \}
\]
\[
- [c_7 x_{sn} - c_7 \cos(x_{sn})]
\]
Define the parameter estimate vector \( \theta = [\theta_1, \theta_2]^T \), where \( \theta_1 = \theta'_1 - \bar{\theta}_1 \) and \( \theta_2 = \theta'_2 - \bar{\theta}_2 \), and let the matrix function be given by
\[
\Psi(x) = \begin{bmatrix} 1 & (x_1 - x_{sn}) \end{bmatrix} \tag{43}
\]

Thus, the perturbed functions will satisfy the matching condition (18). From Theorem 2, we obtained the following adaptive feedback law that solves the adaptive \( L_2 \) disturbance attenuation problem for the system.
\[
\begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 
\end{bmatrix} = \text{Proj} \{ \phi(x) \}
\]
\[\text{where} \quad \phi(x) = \begin{bmatrix}
\rho_1 \left[ b_1 (\cos x_{sn} - \cos x_1) \right] + \frac{c_1}{c_1} (x_2 - x_{sn}) \\
\rho_2 \left[ b_2 (\cos x_{sn} - \cos x_1) \right] + \frac{c_2}{c_2} (x_1 - x_{sn}) (x_1 - x_{sn}) \\
\rho_3 \left[ b_3 (\cos x_{sn} - \cos x_1) \right] + \frac{c_3}{c_3} (x_1 - x_{sn}) (x_1 - x_{sn}) \\
\end{bmatrix}
\]

and \( \rho_1 > 0 \) and \( \rho_2 > 0 \) are the adaptation gains.

### 4.3 Simulation results

The simulation was implemented in the PSASP package which is a professional testing system for power systems designed by Chine Electrical Power Research Institute. A synchronous generator (100MW) against an infinite-bus with SCR excitor was chosen as the example system. Dynamical performance under a fault will be tested for the cases with different disturbance attenuation level \( \gamma \). The fault considered in this simulation is a symmetrical three-phase short-circuit fault during the time period \( 0\sim0.2 \) (sec.), and the fault location is at the middle of the transmission line. Also, the limitation of excitation value \( 0.0 \) (p.u.) \( \leq V(t) \leq 4.0 \) (p.u.) is considered in the simulation.

For the power system, we consider the following operating point:

\[
\delta_0 = 0.7439 \text{ rad.}, \quad \omega_0 = 1.0 \text{ p.u.}, \quad E'_{pu} = 0.9361 \text{ p.u.}
\]

Hence, the desired equilibrium for the closed loop system is given by \( (x_{sn}, 0, x_{sn}) = (0.7439, 0, 0.9361) \). The weighting coefficients in penalty signal are chosen as \( q_1 = q_2 = q_3 = 0.1 \), and the parameter adaptation gains as \( \rho_1 = \rho_2 = 0.01 \). Using these design parameters and nominal parameters of the power system, the adaptive controller with \( L_2 \) disturbance attenuation was designed by Theorem 2. The nominal value of the plant parameter are given as follows
\[M = 12.922 \text{sec.}, \quad D = 0.15 \text{ p.u.}, \quad V_s = 1.0 \text{ p.u.}, \]
\[x_d = 0.8258 \text{ p.u.}, \quad x_d = 1.045 \text{ p.u.}, \quad x_r = 0.0292 \text{ p.u.}, \]
\[x_l = 0.0266 \text{ p.u.} \]

A simulation results with the designed controller are as shown in Fig.2 and Fig.3.

Figure 2 shows the responses of the power angle \( \delta(t) \), the relative speed of the generator \( \omega(t) \) and the transient EMF \( E'_{pu}(t) \) with the adaptive controller when the disturbance attenuation level are chosen by \( \gamma = 10.0, 1.0, 0.1 \), respectively. Figure 3 shows a simulation result when the model parameter used in the controller design is perturbed 50% from the nominal values. The parameter estimation is also shown in Fig. 2, and the disturbance attenuation level is chosen as \( \gamma = 0.1 \). The initial values of the parameter estimate are set as \( \bar{\theta}_1(0) = \bar{\theta}_2(0) = 0.0 \) in all cases.

It can be seen from the simulation that the dynamical performance and the transient stability can be improved by reducing the disturbance attenuation level \( \gamma \). Indeed, the power angle oscillation is damped out rapidly when \( \gamma = 0.1 \), and controller has a better performance when the parameter is perturbed.
V. CONCLUSIONS

In this paper we discussed the $L_2$ control problem for Hamiltonian systems. We have first shown that the $L_2$ gain from disturbance to a penalty signal can be reduced to any level by injection of additional damping, if the penalty signal is defined properly. Then, we presented a methodology to introduce parameter adaptive mechanism for the $L_2$ controller. Finally, the proposed controller was applied to a power system, and effectiveness of the proposed controller were shown by simulation implemented on a professional testing system.

REFERENCES

1. Ortega, R. and M. Spong, “Adaptive Motion Control