AN OPTIMAL TRACKING CONTROL APPROACH TO THE SUSTAINED ACCELERATION CONSTRUCTION IN A FLIGHT SIMULATOR MOTION SYSTEM

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ABSTRACT

The purpose of this research is to solve some problems with the optimal control approach to the design of moving flight simulators. A tracking model of the optimal washout filter is applied in the human vestibular sensation system. It can efficiently limit the translational motion of the platform and also generate sustained acceleration. The simulation results demonstrate that we can minimize the difference between the physiological outputs of the vestibular organs of the actual system and the simulator platform system. In this work, the otolith senses not only the high frequency component of the acceleration, but also the sustained acceleration constructed by the gravity component with the corresponding rate-limited tilt angle in the simulator.

For the tracking model, the actual acceleration input signal is divided into two parts: one is the high frequency component of the acceleration, and the other is the sustained acceleration. Using the tracking methodology, we achieve a high fidelity motion cueing, which shows that the sensing error of the otolith and semicircular canal can be suppressed to below one threshold unit, and that the displacement can also be controlled within the working space of the simulator. Since an actual (generally below 0.8g) simulator can not produce over 1g of sustained acceleration for a high maneuver aircraft, an f-scale is used to adjust the degree of the simulation limitation.

KeyWords: Optimal washout filter, motion cueing simulation, sustained acceleration, otolith and semicircular canal.

I. INTRODUCTION

It is impossible to accomplish limitless translational and rotational movements within the limited working space of a moving flight simulator. If we can achieve the same motion cueing of the vestibular system in a closed cockpit simulator as in an actual system, then high fidelity in the motion cueing simulation can be attained. This can be accomplished by means of the so-called washout filter. Generally, the filter is a high pass filter. However, keeping the acceleration and the angular rate equal to the original sensation during the process of transforming the signal in the actual space into that in the working space of the simulator is a big problem in the design of a washout filter. For the purpose of rotation, the step signal and low frequency component of the actual angular signal can be filtered out by the high pass filter during an attitude change. Because the differential value of the step signal is zero, the differentiation of the high pass filter output signal is almost equal to the differentiation of the actual original signal. However, for translational movement, when the flight time is very long, it is hard to meet the fidelity requirement by only using a high pass washout filter within the limited working space of the simulator. Therefore, sustained acceleration must be achieved to sense the sustained motion part; that is to say, the actual acceleration input signal must be divided into two parts: one is the high frequency component
of acceleration, and the other is the sustained acceleration. The displacement is generated by the high frequency component, and the sustained acceleration is developed by the gravity component, where the corresponding rate-limited tilt angle prevents angular sensation in the simulator.

Recently, numerous washout filter designs have been proposed for motion cueing simulation. Grant and Reid [1] developed a tuning paradigm and captured it within an expert system. This development focused on the classical algorithms developed at the University of Toronto, but the results are relevant to alternative classical and similarly structured adaptive algorithms. Their paper also provided the groundwork required to develop a tuning paradigm. The necessity of this subjective tuning process was defended. Motion cueing error sources within the classical algorithm were revealed, and coefficient adjustments that reduced the errors were presented.

A modification of the coordinated adaptive washout algorithm was proposed by Parrish, et al. [4], and thereafter, it will be referred to as the “adaptive algorithm.” This algorithm uses both first and second order linear washout filters in conjunction with an optimization method that adjusts the filter gains in real time by minimizing the error between the simulated motion platform responses and the actual vehicle signal.

A novel technique is “optimal control,” which was developed by Sivan, et al. [2]. Their algorithm uses higher order filters, prior to real time application, using optimal control methods. This approach incorporates a mathematical model of the human vestibular system and minimizes the sensory error between the actual aircraft and simulated motion platform system. Improved performance was achieved by Telban, et al. [3]. In their paper, an optimal algorithm based on simulated aircraft angular velocity inputs was discussed. Models of the human vestibular system were incorporated within the algorithm in order to constrain vestibular sensation errors. A set of cueing filters was optimized and generated prior to real time implementation. Nahon and Reid [5] compared three algorithms: The classical washout, optimal control, and coordinated adaptive algorithms. The authors considered that most of the algorithms could be massaged and perform reasonably well, and that a more important consideration was the ease with which a given algorithm could achieve high performance levels.

At present, all of the previously presented schemes either have some drawbacks with respect to sustained acceleration or place limitations on the moving flight simulators, and the fidelity of simulations is reduced by the corresponding errors of the threshold of the pilot sensation. For the optimal washout filter presented by Sivan, et al. [2], the generated angle is not large enough to simulate the steady state of the low frequency for the purpose of acceleration, so there exists a remarkable difference between the actual acceleration and the simulated acceleration when the sustained acceleration is combined with the high frequency component of the acceleration. On the other hand, the error signals for the human vestibular system, i.e., the oolith and the semicircular canal do not satisfy the requirement wherein the error signals are kept below the pilot threshold. One threshold unit of the oolith typically corresponds to an acceleration input of 0.47m/s² at an angular frequency of 0.94rad/s, and one threshold unit of the semicircular canal corresponds to an angle input of 1.45deg/s² at an angular frequency of 0.94rad/s. In addition, it can not efficiently limit the translational motion of the platform in modern simulators.

To solve the above problems, we propose an optimal tracking control approach to sustained acceleration construction and a tracking model of the optimal washout filter in the human vestibular sensation system. It can efficiently limit the translational motion of the platform and also generate sustained acceleration. The sustained acceleration is constructed by the gravity component with tilt and a rate-limited angle. The optimal tracking control algorithm used in this work solves the following problems: the sensing error values of the oolith and semicircular canal can not be suppressed below one threshold unit, and the displacement can not be controlled within the working space of the simulator. Finally, this paper compares the performance of the optimal tracking control model and the previous optimal control model.

II. OPTIMAL TRACKING CONTROL APPROACH

2.1 Formulation of the optimal simulation

In the structure of the optimal washout filter, we consider system $S^a$ with input $u^a$ and output $y^a$, and system $S^s$ with input $u^s$ and output $y^s$. The letters $a$ stands for the actual system and the letter $s$ signifies the simulator. We assume that the actual input $u^a$ is given or that it belongs to a known class of signals $U^a$. In order to minimize the error $e$ between the actual system and the simulator, we also presume that the simulator inputs belong to a constraint set $U^s$ which is given. We also let the error $e$ be given as $e = y^i - y^s$. For the given class $U^a$ and the constraint set $U^s$, the optimal simulator design problem is to determine a matrix of linear transfer function $W(s)$ that relates $U^s$ to $U^a$ such that the cost function is minimized. The structure is illustrated in Fig. 1.

Fig. 1. Optimal simulation problem construction.
2.2 The optimal algorithm for the simulator

We shall assume that $S^a$ and $S^s$ are both linear, time invariant, finite dimensional systems, whose state equations are

\[
S^a: \quad \dot{x}^a(t) = A_a x^a(t) + B_a u^a(t), \\
\quad y^a(t) = C_a x^a(t) + D_a u^a(t),
\]

and

\[
S^s: \quad \dot{x}^s(t) = A_s x^s(t) + B_s u^s(t), \\
\quad y^s(t) = C_s x^s(t) + D_s u^s(t),
\]

where $A_a$, $B_a$, $C_a$, and $D_a$ are the parameters of the actual system, $A_s$, $B_s$, $C_s$, and $D_s$ are the parameters of the simulator, and $x^a$ and $x^s$ are the state vectors of the actual and simulated systems, respectively.

Next, we assume that $U^a$ is a class of random processes generated by filtering white noise through a linear system $N$:

\[
N: \quad \dot{x}^n(t) = A_x x^a(t) + B_n n(t), \\
\quad u^n(t) = C_x x^a(t),
\]

where $A_x$, $B_x$, and $C_x$ are the parameters of the noise shaping filter $N$, $x^n$ is the state of this filter, and $n(t)$ is the white noise.

We combine the three linear systems (1), (2), and (3) so as to obtain the following augmented linear system $A$:

\[
A: \quad \dot{x}(t) = \overline{A} x(t) + B u(t) + \overline{H} n(t), \\
\quad e(t) = \overline{C} x(t) + D u(t),
\]

where

\[
\overline{A} = \begin{bmatrix} A_a & 0 & B_a C_x \\ 0 & A_s & 0 \\ 0 & 0 & A_s \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B_a \\ 0 \\ 0 \end{bmatrix}, \quad \overline{H} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
\]

and

\[
\overline{C} = \begin{bmatrix} -C_a & C_s & -D_s C_a \end{bmatrix}, \quad \overline{D} = D_a.
\]

Let us define the cost function $J$ such that $J$ is minimized, where

\[
J = E[e(t)^T Q e(t)] + \rho E[u(t)^T R u(t)].
\]

We assume that $\rho > 0$, $R$ is a positive definite matrix, and $Q$ is a positive semidefinite matrix. The scalar $\rho$ is a Lagrangian multiplier, which can be selected such that the weighted mean-square value of $u(t)$ equals a given positive constant.

In addition,

\[
E[x^T Q e] = E[(\overline{C} x + \overline{D} u)^T Q (\overline{C} x + \overline{D} u)]
\]

\[
= E[x^T \overline{C}^T Q \overline{C} x + x^T \overline{D}^T Q \overline{D} u + u^T \overline{D}^T Q \overline{C} x + u^T \overline{D}^T Q \overline{D} u].
\]

Substituting Eq. (9) into Eq. (8) and rearranging results in

\[
J = E[x(t)^T R_s x(t) + 2 x(t)^T R_s u'(t) + u'(t)^T R_s u'(t)],
\]

where

\[
R_1 = \overline{C}^T Q \overline{C}, \quad R_2 = \overline{C}^T \overline{D} \overline{D}^T Q \overline{C}, \quad R_3 = \rho R + \overline{D}^T \overline{D} Q \overline{C}.
\]

The cost function $J$ of Eq. (11) is minimized when

\[
u = \overline{R}_s^{-1} \overline{B}^T P x,
\]

and can then be defined as

\[
u = u' + \overline{R}_s^{-1} \overline{R}_2 x,
\]

where

\[
u'(t) = -F x(t);
\]

the optimal feedback gain matrix $F$ is defined as

\[
F = \overline{R}_s^{-1} \overline{B}^T P + \overline{R}_2 T
\]

and $P$ is the unique nonnegative definite solution of the following algebraic Riccati equation:

\[
P (\overline{A} - \overline{B} \overline{R}_2^{-1} \overline{R}_2 T) + (\overline{A} - \overline{B} \overline{R}_2^{-1} \overline{R}_2 T)^T P
\]

\[
- (P \overline{B} + \overline{R}_2) \overline{R}_2^{-1} (\overline{B}^T P + \overline{R}_2) + \overline{R}_1 = 0.
\]

By computing the values of $F$, we can get $u'$ corresponding to $u'$. The optimal washout filter should be designed using this methodology. A model of the vestibular system was presented by Sivan, et al. [2] (see Fig. 2). The simulated acceleration, the displacement of the simulator, and the sensing error values of the otolith and semicircular canal can be obtained by applying the design of the optimal algorithm.
From the model of the vestibular system shown in Fig. 2, the input signals \( \dot{\theta}(t) \) and \( \theta(t) \) are used for the actual system \( S^a \) and the simulated system \( S^s \). In the simulator, the input signals come from the output of the optimal washout filter \( W(s) \). Obviously, the optimization process is to design a washout filter such that the difference of motion cueing between the actual system and the simulator is minimized. However, in the actual system, the tilt angle is not constructed to generate the sustained acceleration part, so the optimal washout filter can only simulate the high frequency component of acceleration. Therefore, the displacement of the moving flight simulator will have to exceed the working space in order to achieve the goal of optimization, which will minimize the tracking error and maximize the simulation fidelity. Thus, we propose using the tracking model shown in Fig. 3 for the actual aircraft system to construct the corresponding tilt angle, which will generate the sustained acceleration part by means of the gravity component. The proposed optimal tracking control approach to sustained acceleration construction will be described in detail in the following.

### 2.3 Design of the optimal tracking control

A tracking model for the actual vestibular system \( S^a \) is shown in Fig. 3. This figure shows that the actual acceleration input signal can be divided into two parts: one is the high frequency component of acceleration, and the other is the sustained acceleration. The sustained acceleration is constructed by the gravity component with the corresponding rate-limited tilt angle in the simulator. The constructed tilt angle in the tracking model of the actual system \( S^a \) is tracked by the optimal washout filter \( W(s) \) such that the sustained acceleration can be simulated within the working space, which means that high fidelity motion cueing can be attained in the flight simulator. As shown in Fig. 3, the actual pitch angle is added to the constructed tilt angle for sensing in the semicircular canal system.

\[
\begin{align*}
    & \text{Otoilet} \quad \text{H} \\
    & G = \frac{s + a_k}{s + b_k} \\
    & \text{Yoto(t)} \\
    & -1/g \\
    & g \\
    & \text{Semicircular Canal} \\
    & \text{Yacc(t)} \\
    & G_1 = \frac{s^2}{(\tau_2s + 1)(\tau_2s + 1)} \\
    & G_2 = \frac{s}{(\tau_2s + 1)(\tau_2s + 1)} \\
    & \text{Fig. 3. A tracking model for the actual vestibular system.}
\end{align*}
\]

As shown in Fig. 3, the sustained acceleration is constructed by the gravity component with the corresponding tilt angle \( \theta_{\text{tilt}} = -\sin^{-1}(\dot{\theta}(t)/g) \). Here, we have \( \sin\theta_{\text{tilt}} = \theta_{\text{tilt}} \) for \( \theta_{\text{tilt}} \leq 0.26 \). Obviously, this may be possible for civil aircraft, since the sustained acceleration is less than 1g \((9.8\text{m/s}^2)\). But for highly maneuverable aircraft, it is difficult to construct a tilt angle which corresponds to the more than 1g of sustained acceleration through computation of the tilt angle \( \theta_{\text{tilt}} \). This is a bottleneck for sustained acceleration construction.

If a tilt angle is used to construct the sustained gravity component, we need consider the tilt angular rate. In practice, it is assumed that the angular rate is sensed accurately above a certain threshold of motion perception, and that the practical value of the perception threshold is 6deg/s. This means that one sensing threshold unit is defined as the sensing output signal of the semicircular canal, which corresponds to an angular rate input of 6deg/s at an angular frequency of 0.94rad/s. Thus, in this paper, if the induced tilt angular rate is more than 6deg/s during tilt angle construction, then we need to add a tilt-rate limiter to guarantee that there will be no perception in the semicircular canal. But the fidelity of simulator motion cueing will be reduced in this situation due to the use of the tilt-rate limiter. There is a trade-off between the fidelity and the tilt-rate limit for the current motion-cueing simulator.

From Fig. 3, we can see the reason why the setting is 0.5. This setting will enable the input signal of the otolith to match the value of the original linear acceleration. The parameter of \((-1/g)\) in the figure is regarded as the tilt coordinated parameter that can be used to obtain the constructed tilt angle \( \theta_{\text{tilt}} \). The tilt angle \( \theta_{\text{tilt}} \) is formulated based on the relation \( \theta_{\text{tilt}} = \theta_{\text{tilt}} - \sin^{-1}(\dot{\theta}(t)/g) \times 0.5 \times (-1/g) \times 2 \), which indicates a setting of 2 in front of the semicircular canal. In Fig. 3, the half of the tilt angle that is added to the half of the input roll angle produces the half of the input signal in the semicircular canal. Before it is used in the semicircular canal system, the combined angle has to double because it was multiplied by 0.5 in the former process.

Also, the model of the semicircular canal is formulated as

\[
G_1 = \frac{s^2}{(\tau_2s + 1)(\tau_2s + 1)}
\]

for the angular sensor input. If we use an angular velocity sensor, then the model of the semicircular canal will be modified as

\[
G_2 = \frac{s}{(\tau_2s + 1)(\tau_2s + 1)}.
\]
and the constructed tilt angle must be differentiated into the tilt angular rate before it is added to the input sensing signal of the semicircular canal system.

We shall use a linear model for the otolith, where $S_{p}(t)$ is the specific force and $Y_{otol}(t)$ is the normalized firing rate. Also, $\theta(t)$ is the angular motion, and $Y_{scc}(t)$ is the normalized firing rate for the semicircular canal. The values of the parameters for this model are

$$G_0 = 2.16 (s^2/m), \quad a_0 = 0.076 (rad/s),$$
$$b_0 = 0.19 (rad/s), \quad G_s = 233 (s^2/\text{rad}),$$
$$\tau_1 = 5.9 (s), \quad \tau_2 = 0.003 (s).$$

A simplified block diagram describing the vestibular system is shown in Fig. 3. The state equations of the vestibular system composed of a pilot subjected to the actual motion augmented by the two states $\dot{d}(t)$ and $\ddot{d}(t)$ are, thus,

$$S^o: \dot{x}^o(t) = A^o \dot{x}^o(t) + B^o u^o(t),$$
$$y^o(t) = C^o \dot{x}^o(t) + D^o u^o(t),$$

where the state vector $x^o$ has the following five coordinates: $x_1^o$ is the state of the otolith model, $x_2^o$ and $x_3^o$ are the states of the semicircular canal model, and $x_4^o$ and $x_5^o$ are the displacement $\dot{d}(t)$ and the velocity $\ddot{d}(t)$ of the airplane, respectively, where

$$A^o = \begin{bmatrix} -b_0 & 0 & 0 & 0 & 0 \\ 0 & -a_s & 1 & 0 & 0 \\ 0 & -b_s & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0001 & 1 \\ 0 & 0 & 0 & 0 & -0.0001 \end{bmatrix},$$

$$B^o = \begin{bmatrix} G_0(a_0 - b_0) & -G_0(a_0 - b_0) \\ 0 & -a_s b_s G_s \\ 0 & -b_s^2 G_s \\ 0 & 0 & -1/2g \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$C^o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$D^o = \begin{bmatrix} G_0 -G_0 g \\ 0 & G_s b_s \end{bmatrix},$$

$$x^o(t) = [x_1^o(t) \quad x_2^o(t) \quad x_3^o(t) \quad x_4^o(t) \quad x_5^o(t)].$$

The system $S^o$ is the vestibular system of the pilot in the simulator:

$$S^o: \dot{x}^o(t) = A^o \dot{x}^o(t) + B^o u^o(t),$$
$$y^o(t) = C^o \dot{x}^o(t) + D^o u^o(t),$$

where

$$A^o = \begin{bmatrix} -b_0 & 0 & 0 & 0 & 0 \\ 0 & -a_s & 1 & 0 & 0 \\ 0 & -b_s & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0001 & 1 \\ 0 & 0 & 0 & 0 & -0.0001 \end{bmatrix},$$

$$B^o = \begin{bmatrix} G_0(a_0 - b_0) & -G_0(a_0 - b_0) \\ 0 & -a_s b_s G_s \\ 0 & -b_s^2 G_s \\ 0 & 0 & -1/2g \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$C^o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

with

$$a_s = (\tau_1 + \tau_2)/\tau_1 \tau_2, \quad b_s = 1/\tau_1 \tau_2,$$

$$u^o(t) = \begin{bmatrix} \ddot{d}(t) \\ \phi(t) \end{bmatrix}, \quad y^o(t) = \begin{bmatrix} y_{oto}(t) \\ y_{scc}(t) \end{bmatrix}.$$

In addition, we add two more output signals with the output equation in the simulated system $S^o$:

$$y^o_j(t) = C_d x^o_j(t),$$

where

$$y^o_j(t) = \begin{bmatrix} \ddot{d}(t) \\ \dot{d}(t) \end{bmatrix}, \quad C_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

We assume that both $\ddot{d}$ and $\phi$ are first order stochastic processes:
\[
\ddot{u}(t) = \begin{bmatrix} -\beta_1 & 0 \\ 0 & -\beta_2 \end{bmatrix} u''(t) + \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} n(t),
\]
where
\[
\dot{n}(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix},
\]
\(n_1\) and \(n_2\) are independent white noise processes, and we use the notation
\[
A_w = \begin{bmatrix} -\beta_1 & 0 \\ 0 & -\beta_2 \end{bmatrix}, \quad B_w = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, \quad C_w = I,
\]
where
\(\beta_1 = 1\), \(\beta_2 = 0.1\).

As in Eq. (4), we can obtain a linear system as follows:
\[
\begin{aligned}
A : & \dot{x}(t) = \overline{A}x(t) + \overline{Bu}'(t) + \overline{H}n(t), \\
e(t) = & \overline{C}x(t) + \overline{Du}'(t),
\end{aligned}
\]
where,
\[
x(t) = \text{col}(x^a(t)^T, x'(t)^T, x''(t)^T), \\
y(t) = \text{col}(e(t)^T, y'(t)^T),
\]
The criterion \(J\) to be optimized is selected as
\[
J = E[e(t)^TQe(t) + \rho[u'(t)^TRu' + y'(t)^TDu']].
\]
In Eq. (19) and Eq. (21), the cost function can be rewritten as
\[
J = E[x(t)^TQx(t) + x(t)^T\overline{C}x(t) + \overline{C}u'(t)^TDu' + u'(t)^T\overline{D}u'(t)] =
E[x(t)^TR_1x(t) + 2x(t)^TR_2u'(t)^Tu'(t)^TR_2u'(t)],
\]
where
\[
Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}, \quad \overline{Q} = \begin{bmatrix} Q & 0 \\ 0 & \rho R_D \end{bmatrix}, \\
R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}, \quad \overline{R} = \begin{bmatrix} R_D & 0 \\ 0 & R_D \end{bmatrix},
\]
\[
R_1 = \overline{C}^T\overline{Q}, \quad R_2 = \overline{D}^T\overline{QD}, \quad q_1 = q_2 = 0.707, \quad r_1 = r_2 = 5.77\times10^{-3}, \\
r_2 = 0.707, \quad \rho = 10.
\]
Substituting the results into the algebraic Riccati equation, we obtain
\[
\begin{aligned}
\overline{A}^TP + P\overline{A} - (P\overline{B} + R_2)R_1^{-1}(\overline{B}^TP + R_2^{-1}) + R_1 = 0.
\end{aligned}
\]
Let us write \(F\) in block form corresponding to the blocks of \(x(t)\):
\[
F = [F_1, F_2, F_3]
\]
so that, from Eq. (14), we have
\[
u'(t) = -F_1x^a(t) - F_2x'(t) - F_3x''(t).
\]
Let the matrix \(P\) of Eq. (16) have the block form
\[
P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix},
\]
where
\[
F_1 = R_1^{-1}(\overline{B}^TP_2 - \overline{D}^TQC), \quad F_2 = -F_1, \\
F_3 = R_1^{-1}(\overline{B}^TP_3 - \overline{D}^TQD).
\]
Since we deal only with steady state situations and we assume that \(A\) is stable, we can derive from Eqs. (17) and (18), respectively,
\[
X^a(s) = (sI - A)^{-1}B_u^u(s), \quad X'(s) = (sI - A)^{-1}B_u^u(s),
\]
Substituting Eqs. (27) and (28) into Eq. (25), and rearranging Eq. (25) yields
\[
U'(s) = -(I + F_2(sI - A)^{-1}B_u^u(s))^{-1}(F_1(sI - A)^{-1}B_u^u(s) + F_3X^a(s)).
\]
Since
\[
C_w = I,
\]
where
\[
A = \begin{bmatrix} A_n & 0 & BC_n \\ 0 & A_n & 0 \\ 0 & 0 & A_n \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \overline{H} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
\]
\[
\overline{C} = \begin{bmatrix} -C_w & C_w & -D_wC_w \\ 0 & C_w & 0 \end{bmatrix}, \quad \overline{D} = \begin{bmatrix} D_w \\ 0 \end{bmatrix},
\]
x\((t)\) and \(u'(t)\) are identical for all \(t\), and Eq. (29) reduces to
\[
U'(s) = -(I + F_2(sI - A)^{-1}B_u^u(s))^{-1}(F_1(sI - A)^{-1}B_u^u(s) + F_3X^a(s)).
\]
Thus, the following equation is obtained:

\[ U'(s) = W(s) \cdot U^a(s), \]

where

\[
W(s) = \begin{bmatrix} W_{11}(S) & W_{12}(S) \\ W_{21}(S) & W_{22}(S) \end{bmatrix},
\]

where

\[
W_{11} = 0.0437(s + 0.081)(s + 1.12 \times 10^{-5})(s + 8.78 \times 10^{-6}) \\
\quad \times (s^2 + 0.37s + 0.034)/D(s),
\]

\[
W_{12} = 1.216 \times 10^{-6}(s + 0.17)(s + 0.075)(s + 0.049) \\
\quad \times (s^2 + 2 \times 10^{-5} + 1.537 \times 10^{-10})/D(s),
\]

\[
W_{21} = -0.051(s + 0.316)(s + 0.16)(s + 0.0315) \\
\quad \times (s^2 + 0.472s + 0.082)/D(s),
\]

\[
W_{22} = (s + 0.26)(s + 0.17)(s + 0.15) \\
\quad \times (s^2 + 0.44s + 0.072)/D(s),
\]

\[
D(s) = (s + 0.26)(s + 0.15)(s + 0.081) \\
\quad \times (s^2 + 0.45s + 0.076).
\]

### III. SIMULATION RESULTS AND COMPARISON

Although the optimal washout filter [2] can achieve brilliant performance, the sensing error values of the otolith and semicircular canal still cannot be suppressed below one threshold unit, and the displacement also cannot be controlled within the working space of the simulator. Therefore, the above improved optimal washout filter was adopted to solve these problems. Simulation results were obtained and comparisons made to demonstrate its superiority.

In this simulation, we interpreted the two degrees of freedom as surge linear motion with pitch angular motion. For the actual input signal, an airplane was accelerated forward at 2m/s² for 10s and then flown with a constant speed for another 25s. For the output signal, we compared the improved optimal response with the optimal response which was presented in [2]. The optimal signal was represented as the output signal of the optimal washout filter \(W(s)\) by using the model of the vestibular system shown in Fig. 2, and the improved optimal signal was represented as the output signal of the optimal washout filter \(W(s)\) by using the tracking model of the vestibular system shown in Fig. 3.

Using \(\beta_1 = 1\) and \(\beta_2 = 0.1\) for the first order stochastic process as shown in Eq. (20) and using \(q_1 = q_2 = 0.707, r_1 = r_2 = r_3 = 5.77 \times 10^{-3}, r_2 = 0.707\) and \(\rho = 10\) for the cost function (22), we could obtain the optimal washout filter \(W(s)\) as shown in Eq. (33).

The parameters of the symmetric weighting matrices \(Q, R, R_d\) can be selected by the designer, who bases his choice on the relative importance of the various states and controls. A certain amount of trial and error is usually required with an interactive computer program before a satisfactory design can be obtained. There are, a few guidelines need to be employed.

Figure 4 compares the acceleration output responses for the two kinds of optimal washout filters. Note that the acceleration response of the improved optimal algorithm is almost the same as the actual acceleration input. Conversely, there is an obvious difference between the simulated acceleration of the former optimal algorithm and the actual acceleration input. Figure 5 shows a comparison between the optimal algorithm responses for the displacement of the simulator. Note that the displacement for the former optimal algorithm is over 8m, but for the improved optimal algorithm, the moving range of the simulator is effectively limited to within 1m, which conforms to the requirement of the working space of the modern simulator. Figures 6 to 9 show a comparison of the optimal algorithm responses for the sensing error values of the otolith and semicircular canal. Note that for the improved optimal algorithm, the error values can be kept below one threshold unit. By way of parenthesis, Fig. 8 shows one pulse signal which is over one threshold unit. A pilot cannot sense such a signal rapidly because the sustained time is very short for this pulse signal, so we can ignore its effect.

In Figs. 6 to 9, one threshold unit of the otolith is defined as a sensing signal of translational motion perception, which corresponds to an acceleration input of 0.47m/s² at an angular frequency of 0.94rad/s, and one threshold unit of the semicircular canal is defined as a sensing signal of rotational motion perception, which corresponds to an angular rate input of 6deg/s at an angular frequency of 0.94rad/s for the values of the following parameters in the tracking model:

\[ G_0 = 2.16 \text{ (s}^2/\text{m}), \quad a_0 = 0.076 \text{ (rad/s)}, \]

\[ b_0 = 0.19 \text{ (rad/s)}, \quad G_1 = 233 \text{ (s}^2/\text{rad)}, \]

\[ \tau_1 = 5.9 \text{ (s)}, \quad \tau_2 = 0.003 \text{ (s)}. \]

The parameters of the tracking model are selected from the models of the otolith and semicircular canal.

It is assumed that the linear and rotational acceleration is sensed accurately above certain thresholds of motion perception, and that small acceleration below one threshold unit is not sensed.
IV. DISCUSSION AND FUTURE DEVELOPMENT

The work presented in this paper investigated the optimal tracking for sustained acceleration construction. In the optimal washout filter design, the high frequency part of the input signal is maintained by high pass filter, and the low frequency part is sensed as the sustained acceleration. In order to enable a pilot to sense the sustained acceleration in the flight simulator, it can be constructed with a gravity component with the corresponding rate-limited tilt angle.
When the transient and sustained acceleration components are combined, the motion cueing feels like the actual motion.

We have demonstrated the superiority of the optimal tracking control approach and solved some problems with the previous optimal washout filter. These problems indicated that the sustained acceleration component was too small, and that the sensing error value of the otolith could not be suppressed below one threshold unit. Compared to the previous optimal work, the present simulation results reveal that the sustained acceleration can be constructed with a tilt angle in the improved optimal washout filter design. The complete simulated acceleration signal in the simulator is equal to the actual acceleration in the flight, and the fidelity of the simulation shows that the sensing error values of the otolith and semicircular canal are below one threshold unit, within the working space of the flight simulator. Therefore, the optimal tracking control approach can be applied to sustained acceleration construction to improve the performance of the previous optimal washout filter.

The purpose of this “tilt-coordination” mechanism is to orient the gravity vector in the flight simulator for sustained acceleration generation. However, since the angular velocity associated with tilt-coordination is an artifact generated to trick the pilot’s senses, it is important for the pilot to not be able to sense its presence, which means that the angular rate must be suppressed below one threshold unit of the semicircular canal. It is difficult to perfectly satisfy the requirements for the angular rate and the fidelity of the simulation simultaneously. We think this is a tradeoff problem. By slowly rotating the cabin, the pilot can perceive a component of gravity as linear sustained acceleration. This effect works only if no rotation can be felt and visual inputs (the position of the horizon) remain at the same relative position. The combined effect of linear acceleration of the high frequency component and the perceived linearly constructed sustained acceleration due to rotation will show a dip caused by the maximum allowable rotation velocity. A large dip is very disturbing for the pilot. Therefore, further investigation of the simulator motion-drive problem should focus on the contradiction between the angular rate and the fidelity of motion cueing in linear acceleration simulation.

In this paper, we have compared the performance of the optimal tracking control approach with that of the optimal approach for sustained acceleration construction. For the optimal control algorithm reported by Reid and Nahon, the present results demonstrate the following points.

1. \( W_1(s) \) has the form of a high-pass filter, and \( W_2(s) \) is a low-pass filter, so this channel has the function to tilt-coordination.

2. The aircraft Euler angles are passed through \( W_2(s) \) to produce the simulator Euler angles. \( W_2(s) \) tends to have a unity transfer function in pitch and roll, and acts as a high-pass filter in yaw.

3. The equations coded in the design program enable the produced filters to be numerous and of high order. However, examination of these filters always reveals that extensive simplifications can be made. The order of the optimal filter can usually be reduced by removing some roots from the numerators and denominators of their transfer functions. The order of \( W_1(s) \) is 4 for surge, and the order of \( W_2(s) \) is 5 for pitch/surge.

4. The design algorithm includes a vestibular model that enables it to minimize the pilot’s motion sensation error between the aircraft and simulator. Adjustment of the algorithm can be performed by changing the weights for physically meaningful quantities.

5. Tilt-rate limiting is not included in the algorithm because it has negative effects on the fidelity of simulation.

The superiority of the optimal tracking control approach over the optimal control approach has been demonstrated in terms of the washout filter. The sustained acceleration can be constructed with a tilt angle, the complete simulated acceleration signal is equal to the actual acceleration within the working space of the flight simulator, and the sensing error values of the otolith and semicircular canal are below one threshold unit.

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**REFERENCES**


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