ROBUST ADAPTIVE CONTROL FOR STRICT-FEEDBACK NONLINEAR SYSTEMS

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ABSTRACT

In this paper, we propose a robust adaptive tracking control based on the backstepping strategy for strict-feedback nonlinear systems with non-parametric uncertain nonlinearities. It is shown that one can design a stable adaptive control system provided that the uncertain nonlinearities can be decomposed by unknown bounded nonlinear functions and known nonlinear functions. The proposed method can deal with uncertain nonlinearities that appear at the control input term too. It is also shown that suitable choice of design parameters guarantees the convergence of tracking error to any desired bound.

KeyWords: Adaptive control, nonlinear systems, robust adaptive control, non-parametric uncertainties, backstepping.

I. INTRODUCTION

During the last two decades, a great deal of attention has been attracted to nonlinear control and several significant results have been derived for control of nonlinear systems. The major contributions on these works are ‘feedback linearization’ and ‘passification’ of nonlinear systems in the differential-geometric theory of nonlinear feedback control (see for instance, seminal books [1-3]). Some drawback for the methods, however, appears clearly in the case where there exist uncertainties since the methods are applicable provided that the controlled system is known.

The adaptive control scheme is considered as one of the methods against the uncertainties and to improve the robustness of the control system. With this in mind, many sort of adaptive strategies, based on the above-mentioned geometric theory, for nonlinear controlled systems were proposed in the first of 90’s. Most of them, unfortunately, had some restrictions such as matching condition [4], linear growth condition for nonlinearities [5] and requirement of the knowledge of the so-called ‘control Lyapunov function’ [6].

To break these restrictions, a novel adaptive control scheme called ‘adaptive backstepping method’, which is pioneered by [7], was proposed for systems that be transformable into the so-called ‘parametric feedback form’ [8]. Although the method requires the system be transformable into the parametric feedback form, the method brakes the matching condition and is also applicable to wider class of nonlinearities without the growth condition of nonlinearities. However, the adaptive method is still less restrictive, that is, the method only handles the parametric uncertainties with unknown constant which appears linearly in the system equations. As is well known in adaptive controls for linear systems [9,10], the most adaptive schemes have undesirable robustness properties for unideal situation. It might be important from the point of practical application to consider a more robust adaptive control strategy for nonlinear systems with non-parametric uncertainties in nonlinear terms.

Recently, there are many papers that deal with robust adaptive control problems for parametric feedback nonlinear systems with exogenous disturbances and/or non-parametric uncertain nonlinearities [11-14,16-18]. However, the known nonlinearities that appeared at the control input were still restricted to [11-13]. Although the method in [14,18] solves the control problem for systems with unknown control directions, the uncertainties that appeared at the control input were basically constants in [14] and the matching condition is imposed in the method in [18]. The
methods provided in [16] and [17] are new strategies that combine the neural network (NN) control and backstepping method. However, the method in [16] is restricted to systems with constant uncertainties at control input. The method in [17] can handle more general uncertainties, however in order to apply the NN approximation, some additional assumptions for uncertainties, such as the uncertainties are continuous and the derivatives of uncertainties are bounded, are imposed.

In this paper, we propose a robust adaptive tracking control based on the backstepping strategy for strict-feedback nonlinear systems with additive and multiplicative non-parametric uncertain nonlinearities. That is, we consider the case where the controlled system has non-parametric uncertainties, which is not necessarily continuous, at control input (virtual input). It is shown that if the uncertain nonlinearities can be divided to a known function and an unknown but bounded nonlinearity, then one can design a stable adaptive control system. It is also shown that a suitable choice of design parameters guarantees the convergence of tracking error to any desired bound. Further the effectiveness of the proposed method will be confirmed through a numerical simulation for a magnetic levitation ball system model.

II. PROBLEM STATEMENT

We consider in this paper single-input/single-output nonlinear systems in the following strict-feedback form [8]

\[
\begin{align*}
\dot{x}_1 &= f_0(x, t) + g_0(x, t) \dot{g}_1(x_1) x_2 \\
\dot{x}_2 &= f_1(x, t)f_2(x, x_2) + g_2(x, t) g_3(x_1, x_2) x_3 \\
& \vdots \\
\dot{x}_n &= f_n(x, t)f_n(x_1, \ldots, x_n) + g_n(x, t) g_m(x_1, \ldots, x_n) u \\
y &= x_1
\end{align*}
\]

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) is the \( n \)-dimensional state vector, \( y, u \in \mathbb{R} \) are the scalar output and control input. \( f_0, g_0 \) are unknown nonlinearities and the functions \( f_n, g_n \) are sufficiently smooth known nonlinear functions. In the conventional adaptive backstepping methods, it is assumed that there are no non-parametric uncertainties in the input (virtual input) term, \( i.e., \) only the case where there are unknown virtual input coefficients has been considered. Here, we consider the case where there are non-parametric uncertainties as in (1).

We make the following assumptions on the nonlinear system (1).

Assumptions.

(A-1) The unknown nonlinearity \( f_0(x, t) \) is bounded for all \( t, x \). That is there exists a positive constant \( \widetilde{f}_0 \) such that

\[ |f_0(x, t)| \leq \widetilde{f}_0 \]

(A-2) The unknown nonlinearity \( g_0(x, t) \) is bounded and the sign is known for all \( t, x \). Without loss of generality, we assume that the sign of \( g_0 \) is positive. That is there exists a positive constant \( \bar{g}_0 \) such that

\[ 0 < g_0(x, t) \leq \bar{g}_0 \]

(A-3) There exists a known function \( g_{1m} \) such that

\[ 0 < g_{1m}(x_1, x_2, \ldots, x_n) \leq g_0(x, t) \]

(A-4) The known function \( g_i(x_1, x_2, \ldots, x_n) \) is non-zero for all \( (x_1, x_2, \ldots, x_n) \).

The objective of this paper is to find an adaptive controller that ensures the output \( y \) track the given reference signal \( y_r \) with the goal:

\[ \lim_{t \to \infty} |y - y_r| \leq \delta \]  \hspace{1cm} (2)

where \( y_r \) is a smooth signal such that

\[ |y_r(t)| \leq d_i, \quad \forall t \in [0, \infty) \]

for some \( d_i \) and \( i = 0, \ldots, n \).

Remark 1. Assumptions (A-2) and (A-3) imposed on the controlled system means that there exist constants \( \bar{g}_0 \) and functions \( g_{1m}(x_1, x_2, \ldots, x_n) \) such that the uncertainties

\[ g(x, t) = g_0(x, t) g_1(x_1) \]

can be evaluated by

\[ g_{1m}(x_1, x_2, \ldots, x_n) g_1(x_1) \leq g_0(x, t) \leq \bar{g}_0 g_1(x_1) \]

This assumption seems similar to the one given in [14]. In [14], however, it is assumed that the uncertainties \( g_1(a_1 x_2, t), g_3(a_2 x_3, t), \ldots, g_n(a_n u, t) \) that appeared at control input (virtual input) can be evaluated as follows:

There exist unknown constant parameters \( l_i > 0 \) and \( \Delta_i^+ \geq \Delta_i^- \) such that for all \( (x_1, x_2, \ldots, u, t) \)

\[ l_i(a_i x_i) + \Delta_i^- \leq g_i(a_i x_i) \leq l_i(a_i x_i) + \Delta_i^+ \]

\[ 1 \leq i \leq n - 1 \]

\[ l_i(a_i u) + \Delta_i^- \leq g_i(a_i x_i, t) \leq l_i(a_i u) + \Delta_i^+ \]

where \( a_i \) are scaled (in the sense \( |a_i| = 1 \)) constant parameters.

It should be noted that this assumption in [14] is quite different from the assumption imposed in this paper. Under the assumption in [14], the method given in [14] can deal with an uncertainty with unknown control directions, however, the method can not deal with non-parametric uncertainties as stated in this paper. This is easily confirmed as follows: Considering the case where \( \Delta_i^+ = \Delta_i^- = 0 \) and the control direction is known, the assumption in [14] means that there are only parametric uncertainties at the control input (virtual input). Since the bounded uncertainties represented by \( \Delta \) can be considered bounded disturbances, the result in [14] is fundamentally obtained under the assump-
tion that the uncertainties that appeared at the control input (virtual input) are parametric uncertainties, even though the method can deal with unknown control directions. In the other words, the assumption in [14] requires that \(|g_i (a_i x_i + 1, t) - l_i(a_i x_{i+1})|\) should be bounded for all \(x_{i+1}\) and \(t\). Thus one can not deal with the uncertainties stated in this paper under the assumption in [14].

III. ROBUST ADAPTIVE CONTROLLER DESIGN

In this section, we provide a robust adaptive controller design based on the backstepping strategy for systems represented in (1) under assumptions (A-1) to (A-4).

Step 1. Set a new variable \(z_i = x_i - y_i = y_i - y_i\) as the tracking error. We design the first virtual input \(D_1\) for \(x_2\) in the \(z_2\)-equation by

\[
\alpha_i = \frac{\eta_i}{g_{10} z_{i+1}} + u_{R12}
\]

where \(c_i\) is any positive constant and \(\tilde{g}_{10}\) is the estimated value of \(g_{10}\), the upper bound of \(g_{10}\), whose adjusting law is to be determined later at the final step so as to be \(\tilde{g}_{10} > 0\). \(u_R11\) and \(u_R12\) are the robust control terms given by

\[
u_R11 = -\tilde{f}_{10} f_{11} \phi_{g_{11}}(\eta_{g_{11}}), \quad \eta_{g_{11}} = f_{11} z_i
\]

\[
u_R12 = -2 \frac{\tilde{g}_{11} \eta_{g_{11}}}{\tilde{g}_{10} \eta_{g_{11}}} \eta_{g_{11}} = \eta_1 z_i
\]

where \(\tilde{f}_{10}\) is the estimated value of \(f_{10}\), whose adjusting law is to be determined later at the final step, and \(\phi_{g_{11}}\) and \(\phi_{g_{12}}\) are given in the following manner with any positive constants \(\delta_{g_{11}}\) and \(\delta_{g_{21}}\).

\[
\phi_{g_{11}}(\eta_{g_{11}}) = \begin{cases} \text{sgn}(\eta_{g_{11}}) & \text{if } |\eta_{g_{11}}| > \delta_{g_{11}} \\ \hat{\phi}_{g_{11}}(\eta_{g_{11}}) & \text{if } |\eta_{g_{11}}| \leq \delta_{g_{11}} \\ 0 & \text{if } \delta_{g_{11}} > 0 \\ -1 & \text{if } \eta_{g_{11}} < -\delta_{g_{11}} \\ 1 & \text{if } \eta_{g_{11}} > \delta_{g_{11}} \end{cases}
\]

\[
\phi_{g_{21}}(\eta_{g_{21}}) = \begin{cases} \text{sgn}(\eta_{g_{21}}) & \text{if } |\eta_{g_{21}}| > \delta_{g_{21}} \\ \hat{\phi}_{g_{21}}(\eta_{g_{21}}) & \text{if } |\eta_{g_{21}}| \leq \delta_{g_{21}} \\ 0 & \text{if } -\delta_{g_{21}} \leq \eta_{g_{21}} \leq 0 \\ -1 & \text{if } \eta_{g_{21}} < -\delta_{g_{21}} \end{cases}
\]

where \(\hat{\phi}_{g_{11}}\) and \(\hat{\phi}_{g_{21}}\) are functions of \(\eta_{g_{11}}\) and \(\eta_{g_{21}}\), respectively, such that \(\phi_{g_{11}}\) and \(\phi_{g_{21}}\) are the class of \(C^\infty\) (i is the number of step, i.e. in this step \(i = 1\)) and

\[
\begin{align*}
\begin{cases}
0 < \phi_{g_{11}}(\eta_{g_{11}}) & \leq 1 & \text{if } 0 < \eta_{g_{11}} \leq \delta_{g_{11}} \\
-1 \leq \phi_{g_{21}}(\eta_{g_{21}}) & < 0 & \text{if } -\delta_{g_{21}} \leq \eta_{g_{21}} < 0 \\
-1 \leq \phi_{g_{11}}(\eta_{g_{11}}) & < 0 & \text{for } -\delta_{g_{11}} \leq \eta_{g_{11}} \leq 0
\end{cases}
\end{align*}
\]

Remark 2. For example, as candidates of such functions \(\phi_{g_{11}}(\eta_{g_{11}})\) and \(\phi_{g_{21}}(\eta_{g_{21}})\), one can consider the following functions:

\[
\begin{align*}
\phi_{g_{11}}(\eta_{g_{11}}) &= \sin \left( \frac{\pi}{2 \delta_{g_{11}}}, \eta_{g_{11}} \right), \quad \text{for } \phi_{g_{11}}(\eta_{g_{11}}) \in C^1 \\
\phi_{g_{21}}(\eta_{g_{21}}) &= \frac{1}{2} \sin \left( \frac{\pi}{2 \delta_{g_{21}}}, \eta_{g_{21}} \right) - \frac{1}{2}, \\
& \quad \text{for } \phi_{g_{21}}(\eta_{g_{21}}) \in C^1
\end{align*}
\]

we also define the following new variables as tuning functions for \(\phi_{g_{11}}(\eta_{g_{11}})\) and \(\phi_{g_{21}}(\eta_{g_{21}})\) which make \(\phi_{g_{11}}(\eta_{g_{11}})\) and \(\phi_{g_{21}}(\eta_{g_{21}})\) into \(C^2\) and \(C^2\), see Appendix 1.

\[
\begin{align*}
\phi_{g_{11}}(\eta_{g_{11}}) &= \sin \left( \frac{\pi}{2 \delta_{g_{11}}}, \eta_{g_{11}} \right), \quad \text{for } \phi_{g_{11}}(\eta_{g_{11}}) \in C^2 \\
\phi_{g_{21}}(\eta_{g_{21}}) &= \frac{1}{2} \sin \left( \frac{\pi}{2 \delta_{g_{21}}}, \eta_{g_{21}} \right) - \frac{1}{2}, \\
& \quad \text{for } \phi_{g_{21}}(\eta_{g_{21}}) \in C^2
\end{align*}
\]

For further details about determining functions \(\phi_{g_{11}}(\eta_{g_{11}})\) and \(\phi_{g_{21}}(\eta_{g_{21}})\) into \(C^2\) and \(C^2\), see Appendix 1.

Consider the following positive definite function:

\[
V_1 = \mu_1 \left\{ \frac{1}{2} \zeta_i^2 + \frac{1}{2} f_{11} (\tilde{f}_{10} - f_{10})^2 + \frac{1}{2} g_{11} (\tilde{g}_{10} - g_{10})^2 \right\}
\]

with any positive constants \(\mu_1, \gamma_{f_1}\) and \(\gamma_{g_1}\). The time derivative of \(V_1\) along the trajectories of \(z_1\)-system yields
\[
\dot{V}_1 = \mu_1 z_1 (f_{10} f_{11} + g_{10} g_{11} (\alpha_1 + z_2) - y_r) \\
+ \mu_1 (\dot{f}_{10} - \overset{\cdot}{f}_{10}) \tau_{f1} + \mu_1 (\dot{g}_{10} - \overset{\cdot}{g}_{10}) \tau_{g11} \\
+ \frac{\mu_1}{\gamma_{f1}} (\dot{f}_{10} - \overset{\cdot}{f}_{10}) (\dot{f}_{10} - \gamma_{f1} \tau_{f11}) \\
+ \frac{\mu_1}{\gamma_{g11}} (\dot{g}_{10} - \overset{\cdot}{g}_{10}) (\dot{g}_{10} - \gamma_{g1} \tau_{g11})
\]  
(14)

where \(z_2 = x_2 - \alpha_1\). From the structure of \(\alpha_1\) in (3) and (4), we have

\[
\dot{V}_1 = -\mu_1 c_1 z_1^2 + \mu_1 g_{10} g_{11} z_1 z_2 + \mu_1 R_{f1} + \mu_1 R_{g1}
\]

Further if \(g_{10}\) is a constant, by setting the robust control term \(u_{R12}\) and the tuning function \(\tau_{g11}\) as

\[
u_{R12} = 0, \quad \tau_{g11} = \eta_{g11}
\]

one can have \(R_g = 0\) for (17). This is considered as an ordinary method for systems with parametric uncertain nonlinearities. However in the case where there exist non-parametric uncertainties, the stability of the control system can not be guaranteed by the ordinary method.

**Remark 4.** It should be noted that in the case where there are no non-parametric uncertainties in the input term we can design the virtual input with \(u_{R12} = 0\). This type of control design schemes have been provided by \([11,12]\)

**Step i (2 \leq i \leq n - 1):** In the step \(i\), we design the virtual input \(\alpha_i\) for \(x_{i+1}\) in \(z_i - x_i - \alpha_i\), as follows:

\[
\alpha_i = \frac{\eta_i}{\bar{g}_{i0}} + u_{R2i}
\]  
(20)

\[
\eta_i = -c_i z_i + \beta_{i-1} + u_{Ri1}
\]

\[
+ \sum_{k=1}^{i-1} \frac{\partial \alpha_j}{\partial x_k} (\gamma_{jk} \tau_{gj,k+1} - \sigma_{jk} \overset{\cdot}{g}_{jk}) + v_i
\]  
(21)

\[
\beta_{j+1} = \sum_{k=0}^{i-1} \frac{\partial \alpha_j}{\partial x_k} z_{j+k}
\]  
(22)

where \(c_i, \gamma_{jk}, \sigma_{jk}\) and \(\sigma_{jk}\) are any positive constants, and \(\bar{g}_{i0}\) and \(\overset{\cdot}{f}_{i0}\) are the estimated value of \(\bar{g}_{i0}\) and \(\overset{\cdot}{f}_{i0}\) respectively, whose adjusting laws are to be determined later at the final step subject to \(\bar{g}_{i0} > 0\). Further \(u_{Ri1}, u_{Ri2}\) are robust control terms given by

\[
u_{Ri1} = -\overset{\cdot}{f}_{i0} f_{i1} \phi_{Ri1} (\eta_{Ri1})
\]

\[
- \sum_{k=1}^{i-1} \overset{\cdot}{f}_{i0} \phi_{Ri1} (\eta_{Ri1})
\]  
(23)

\[
\eta_{Ri1} = f_{i1} z_i
\]
\[ \eta_{Rk,i+1} = f_k \frac{\partial \alpha_{k-1}}{\partial x_l} z_i, \quad k = 1, \ldots, i-1 \]

\[ \eta_{Rk,i+2} = g_k \frac{\partial \alpha_{k-1}}{\partial x_l} x_{k+1} z_i, \quad k = 1, \ldots, i-2 \]

\[ u_{gi} = -2 \phi_{gi0}(\eta_{gi0}) \eta_l \]

\[ \eta_{gi0} = \eta_l z_i \]  (24)

and \( \tau_{Rk,i-1}, \tau_{Rk,i+1} \) (tuning function for \( \hat{f}_{k0} \) and \( \hat{g}_{k0} \), \( k = 1, \ldots, i-1 \)) and \( v_l \) are given by

\[ \tau_{Rk,i-1} = \tau_{Rk,i-2} + \frac{\mu_k}{\mu_i} \eta_{Rk,i-1} \Phi_{Rk,i-1}(\eta_{Rk,i-1}) \]  (25)

\[ \tau_{Rk,i+1} = \tau_{Rk,i+2} + \frac{\mu_k}{\mu_i} \eta_{Rk,i+1} \Phi_{Rk,i+1}(\eta_{Rk,i+1}) \]  (26)

\[ v_l = 0 \]  (27)

\[ v_l = \sum_{k=1}^{i-2} \left\{ \gamma_{ki} \left( \sum_{k=1}^{i-2} \mu_{k+1} \frac{\partial \alpha_k}{\partial f_{k0}} z_{k+1} \right) \right\} \]

\[ + \sum_{k=1}^{i-2} \left\{ \gamma_{ki} \left( \sum_{k=1}^{i-2} \mu_{k+1} \frac{\partial \alpha_k}{\partial g_{k0}} z_{k+1} \right) \right\} \]

\[ + \frac{1}{\mu_l} g_{k1} \frac{\partial \alpha_{k-1}}{\partial x_l} x_{k+1} \Phi_{Rk,i-1}(\eta_{Rk,i-1}) \]  (28)

for \( i = 3, \ldots, n-1 \)

where \( \mu_i \) is any positive constant. We also define the following new variables as tuning functions for \( \hat{f}_{k0} \) and \( \hat{g}_{k0} \):

\[ \tau_{fi} = \eta_{Rf1} \Phi_{Rf1}(\eta_{Rf1}) \]  (29)

\[ \tau_{gi} = \eta_{Rg1} \Phi_{Rg1}(\eta_{Rg1}) \]  (30)

\[ \eta_{Rg1} = 1 \frac{\eta_l}{g_{k0}} z_i \]

Since the time derivative of \( \alpha_{k-1} \) can be represented by

\[ \dot{\alpha}_{k-1} = \beta_{k-1} + \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial x_l} (f_{k0} f_{k1} + g_{k0} g_k x_{k+1}) + \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial \hat{f}_{k0}} \dot{f}_{k0} + \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial \hat{g}_{k0}} \dot{g}_{k0} \]  (31)

we have the following \( \dot{z}_i \) equation:

\[ \dot{z}_i = f_{k0} f_{k1} + g_{k0} g_k x_{k+1} - \beta_{k-1} \]

\[ - \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial x_l} (f_{k0} f_{k1} + g_{k0} g_k x_{k+1}) - \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial \hat{f}_{k0}} \dot{f}_{k0} - \sum_{l=1}^{i-2} \frac{\partial \alpha_{k-1}}{\partial \hat{g}_{k0}} \dot{g}_{k0} \]  (32)

Here define the following positive definite function:

\[ V_i = V_{i-1} + D_i \left( \frac{1}{2} \dot{f}_{k0}^2 + \frac{1}{2} \dot{g}_{k0}^2 \right) \]

\[ + \frac{1}{2} \left( \dot{f}_{k0} - \frac{\dot{f}_{k0}}{\hat{f}_{k0}} - \gamma_{fi} \tau_{fi,i+1} \right)^2 + \frac{1}{2} \left( \dot{g}_{k0} - \frac{\dot{g}_{k0}}{\hat{g}_{k0}} - \gamma_{gi} \tau_{gi,i+1} \right)^2 \]  (33)

with positive constants \( \mu_i, \gamma_{fi} \) and \( \gamma_{gi} \).

The time derivative of \( V_i \) can be evaluated as follows by using the same procedure as in step 1.

\[ \dot{V}_i \leq \sum_{k=1}^{i-1} \left( -\mu_k c_k \dot{z}_i^2 + \mu_k D_k \right) \]

\[ + \sum_{k=1}^{i-1} \frac{\mu_k}{\mu_l} g_{k1} \frac{\partial \alpha_{k-1}}{\partial x_l} x_{k+1} \Phi_{Rk,i-1}(\eta_{Rk,i-1}) \]

\[ + \sum_{k=1}^{i-1} \frac{\mu_k}{\mu_l} g_{k1} \frac{\partial \alpha_{k-1}}{\partial g_{k0}} z_{k+1} \]

\[ + \frac{1}{\mu_l} g_{k1} \frac{\partial \alpha_{k-1}}{\partial x_l} x_{k+1} \Phi_{Rk,i-1}(\eta_{Rk,i-1}) \]

\[ \times \left( \dot{f}_{k0} - \gamma_{fi} \tau_{fi,i+1} + \sigma_{fi} \dot{f}_{k0} \right) \]

\[ \times \left( \dot{g}_{k0} - \gamma_{gi} \tau_{gi,i+1} + \sigma_{gi} \dot{g}_{k0} \right) \]

\[ - \sum_{k=1}^{i-1} \frac{\mu_k}{\mu_l} g_{k1} \frac{\partial \alpha_{k-1}}{\partial x_l} x_{k+1} \Phi_{Rk,i-1}(\eta_{Rk,i-1}) \]

\[ \times \left( \dot{f}_{k0} - \gamma_{fi} \tau_{fi,i+1} + \sigma_{fi} \dot{f}_{k0} \right) \]

\[ \times \left( \dot{g}_{k0} - \gamma_{gi} \tau_{gi,i+1} + \sigma_{gi} \dot{g}_{k0} \right) \]
\[- \sum_{i=1}^{\frac{1}{l+1}} \mu_i \frac{\partial \alpha_i}{\partial \mathbf{x}^i} z_i \left( \mathbf{g}^i_{j_0} - \gamma_{ij} \mathbf{t}_{g_{ij},i+1} + \sigma_{ij} \mathbf{g}^i_{j_0} \right) + \sum_{j=1}^{n} \mu_j v_j z_j \right] \] 

(34)

where

\[ D_k = \sum_{k=1}^{m} \mathbf{f}_{j_0}^i \delta_{R^k},i+1 + \sum_{k=1}^{m} \mathbf{g}_{j_0}^i \delta_{R^k,i+1} + \max(2\delta_1, \mathbf{G}^1) \delta_{R^k,i+1} \]

From the definition of \( \tau_{gk,i+1} \) and \( \tau_{gk,i+1} \) for \( k = 1, \ldots, i-1 \) given in (25) and (26), we have

\[ \tau_{gk,i+1} = \tau_{gk,i+1} + \sum_{j=k+1}^{i} \mu_j f_{n+1} \mathbf{f}_{g_{j-1},i+1} (\mathbf{n}_{g_{j-1},i+1}) z_j \]

(35)

\[ \tau_{gk,i+1} = \tau_{gk,i+1} + \sum_{j=k+1}^{i} \mu_j g_{n+1} \mathbf{f}_{g_{j-1},i+1} (\mathbf{n}_{g_{j-1},i+1}) z_j \]

(36)

It follows from (35) and (36) that the fourth and sixth terms of the right hand side in (34) can be represented as

\[ \sum_{j=1}^{n} \mu_j \frac{\partial \alpha_j}{\partial \mathbf{x}^j} z_j \left( \mathbf{f}_{j_0}^i - \gamma_{ij} \mathbf{t}_{g_{ij},i+1} + \sigma_{ij} \mathbf{f}_{j_0}^i \right) \]

(37)

and

\[ \sum_{j=1}^{n} \mu_j \frac{\partial \alpha_j}{\partial \mathbf{x}^j} z_j \left( \mathbf{g}_{j_0}^i - \gamma_{ij} \mathbf{t}_{g_{ij},i+1} + \sigma_{ij} \mathbf{g}_{j_0}^i \right) \]

(38)

Finally, we obtain from (28), (34), (37), and (38) that

\[ \dot{y}_i \leq \sum_{i=1}^{\frac{1}{l+1}} \mathbf{u}_i \mathbf{c}_i z_i^2 + \mathbf{u}_{e2} \]

(39)

**Step n.** This step is the final step. We design the control input and parameter adjusting laws for \( \dot{\mathbf{f}}_{i_0}^i \) and \( \dot{\mathbf{g}}_{i_0}^i \) in this step.

The control input is designed by

\[ u = \frac{\eta_u}{\mathbf{f}_{i_0}^i} + \eta_{e2} \frac{\mathbf{f}_{i_0}^i}{\mathbf{g}_{i_0}^i} \]

(40)

where \( \eta_u \) and \( \eta_{e2} \) are designed by setting \( i = n \) in (21) to (28) in step i. The parameter adjusting laws for \( \dot{\mathbf{f}}_{i_0}^i \) and \( \dot{\mathbf{g}}_{i_0}^i \), \( i = 1, \ldots, n \), are given as follows:

\[ \dot{\mathbf{f}}_{i_0}^i = \left\{ \begin{array}{ll} 0, & \text{if } U_{\mu}(z) \leq \delta_z \\ \gamma_{ij} \mathbf{t}_{g_{i+1},i+1} - \sigma_{ij} \mathbf{f}_{i_0}^i, & \text{if } U_{\mu}(z) > \delta_z \end{array} \right. \]

(41)

\[ \dot{\mathbf{g}}_{i_0}^i = \left\{ \begin{array}{ll} 0, & \text{if } U_{\mu}(z) \leq \delta_z \\ \gamma_{ij} \mathbf{t}_{g_{i+1},i+1} - \sigma_{ij} \mathbf{g}_{i_0}^i, & \text{if } U_{\mu}(z) > \delta_z \end{array} \right. \]

(42)

where \( \tau_{g_{i+1},i+1} \) and \( \tau_{g_{i+1},i+1} \) has been given in (25), (26), (29), and (30) with \( k = i \) and \( n \). \( \delta_z \) is any positive constant and \( U_{\mu}(z) \) is defined by

\[ U_{\mu}(z) = \frac{1}{2} \sum_{i=1}^{n} \mu_i z_i^2 \]

(43)

Since \( U_{\mu}(z) \geq 0 \) for all \( t \geq 0 \), it is easy to confirm that \( \dot{\mathbf{f}}_{i_0}^i > 0 \) for all finite time \( t \) by setting any positive initial value in the parameter adjusting law (42).
IV. STABILITY AND CONVERGENCE ANALYSIS

In this section, we analyze the stability of the resulting control system and convergence property of the tracking error.

For the case where $U_P(z) > G z$, by defining the positive definite function:

$$V_n = V_{n-1} + \mu_n \left\{ \frac{1}{2} z_n^2 + \frac{1}{2} \gamma \left( \hat{f}_{w0} - f_{w0} \right)^2 \right\}$$

we have from (39), (41), and (42) that

$$\dot{V}_n < \sum_{k=1}^{n} \left[ -\mu_k c_k z_k^2 + \mu_k D_k \right] - \sum_{k=1}^{n} \frac{\mu_k}{2Y_{rk}} \sigma_{rk} \left( \hat{f}_{k0} - f_{k0} \right)^2$$

$$- \sum_{k=1}^{n} \frac{\mu_k}{2Y_{rgk}} \sigma_{rgk} \left( \hat{g}_{k0} - g_{k0} \right)^2$$

$$+ \sum_{k=1}^{n} \frac{\mu_k}{2Y_{rk}} \sigma_{rk} \left( \hat{f}_{k0} - f_{k0} \right)^2 + \sum_{k=1}^{n} \frac{\mu_k}{2Y_{rgk}} \sigma_{rgk} \left( \hat{g}_{k0} - g_{k0} \right)^2$$

we have from (45) that

$$\dot{V}_n < -\alpha V_n + \beta$$

By setting

$$\alpha = \min(2c_k, \sigma_{rk}, \sigma_{rgk})$$

$$\beta = \sum_{k=1}^{n} \mu_k D_k + \frac{\mu_k}{2Y_{rk}} \sigma_{rk} \left( \hat{f}_{k0} - f_{k0} \right)^2 + \frac{\mu_k}{2Y_{rgk}} \sigma_{rgk} \left( \hat{g}_{k0} - g_{k0} \right)^2$$

we have from (47) that

$$\dot{V}_n < -\alpha V_n + \beta$$

Now, we introduce the following continuous function $V$:

$$V = \delta_z + \sum_{k=1}^{n} \mu_k \left\{ \frac{1}{2Y_{rk}} \Delta f_{rk}^2 + \frac{1}{2Y_{rgk}} \Delta g_{rgk}^2 \right\}$$

if $z \in \Omega_0$

$$V_n, \quad \text{if} \quad z \in \Omega_1$$

where $\Delta f_{rk} = \hat{f}_{rk} - f_{rk}, \Delta g_{rgk} = \hat{g}_{rgk} - g_{rgk}, z = [z_1, z_2, \ldots, z_n]^T$ and

$$\Omega_0 = \{ z \in \mathbb{R}^n | U_P(z) \leq \delta_z \}$$

$$\Omega_1 = \{ z \in \mathbb{R}^n | U_P(z) > \delta_z \}$$

It follows that $\dot{V} \leq 0$ for $V \geq \frac{\beta}{\alpha}$. This implies that $V$ is bounded, i.e. $z, \Delta f_{rk}, \Delta g_{rgk}$ are bounded.

Next we analyze the convergence of the tracking error and show the boundedness of all the signals in the control system under the assumption that $\delta_z$ is set such as $\delta_z \geq \frac{\beta}{\alpha}$.

In the case where $\delta_z \geq \frac{\beta}{\alpha}$, we have for $z$ in $\Omega_1$ that

$$\dot{V} \leq -\alpha \delta_z + \beta \leq -\gamma_z$$

where $\gamma_z = \alpha \delta_z - \beta > 0$, and for $z$ in $\Omega_0$, we have

$$\dot{V} = 0$$

Let suppose that there exists a time $t_0$ such that $U_P(z) > \delta_z$ for all $t \geq t_0$. This implies that $V \geq \delta_z$.

We however have from (48) and (49) that

$$\dot{V} = \dot{V}_n \leq -\alpha V_n + \beta$$

$$\leq -\alpha \delta_z + \beta = -\gamma_z < 0$$

where $\gamma_z = \alpha \delta_z - \beta > 0$. It follows that

$$V(t) = V(t_0) + \int_{t_0}^{t} \dot{V}(\tau) d\tau \leq V(t_0) - \gamma_z(t-t_0)$$

Since the right-hand side of (53) will eventually become negative as $t \to \infty$, the inequality contradicts the assumption that $V \geq \delta_z$, and this means that the interval $(t_0, t_1)$ in which $z \in \Omega_0$ should be finite. Let $t_2 \geq t_1$ be a finite time at which $z$ again lies on the boundary of $\Omega_1$, i.e. $z \in \Omega_0$. From the fact that $U_P(z(t_0)) = U_P(z(t_2)) = \delta_z$ and

$$\dot{V} \leq -\gamma_z < 0, \quad \text{for} \quad z \in \Omega_0,$$

it follows that

$$\sum_{k=1}^{n} \mu_k \left\{ \frac{1}{2Y_{rk}} \Delta f_{rk}^2(t_2) + \frac{1}{2Y_{rgk}} \Delta g_{rgk}^2(t_2) \right\}$$

$$< \sum_{k=1}^{n} \mu_k \left\{ \frac{1}{2Y_{rk}} \Delta f_{rk}^2(t_0) + \frac{1}{2Y_{rgk}} \Delta g_{rgk}^2(t_0) \right\}$$

and hence the parameter error decreases a finite amount every time $z$ leaves $\Omega_0$ and reenters. Finally, we can conclude that there exists a time $T$ such that the parameter error converges to a constant for all $t \geq T$ [15]. This means that $U_P(z) \leq \delta_z$ for all $t \geq T$ and it follows in this case that
\[ \lim_{t \to \infty} z_t^2 \leq \frac{2}{\mu_1} \delta_z . \] (54)

Furthermore, since there exists a finite time \( T \) such that \( z \in \Omega_{\epsilon_0} \) for all \( t \geq T \), from (42) and the fact that \( \frac{\dot{z}}{\bar{G}_r} > 0 \) for all finite time \( t \) with any positive initial value, we can conclude that \( \frac{\dot{z}}{\bar{G}_r} > 0 \) for all \( t \). Thus, since \( z \), \( \Delta \bar{f} \) and \( \Delta \bar{g} \) are bounded and \( \frac{\dot{z}}{\bar{G}_r} > 0 \) for all \( t \), it follows that all the signals in the control system are bounded.

We then have the following theorem.

**Theorem.** Under assumptions (A-1) to (A-4), all the signals in the resulting control system with control input (40) and adjusting laws (41) and (42) are bounded and the tracking error \( \varepsilon_t \) converges to a given bound \( |\varepsilon_t|^2 \leq \frac{2}{\mu_1} \delta_z \) as \( t \to \infty \) provided that \( \delta_z \) is set such as \( \delta_z > \frac{\beta}{\alpha} \).

**Remark 5.** For any given positive constant \( \delta_z \), there exist appropriate design parameters \( \gamma_{\tilde{g}}, \gamma_{\tilde{g}}, \) and \( \mu_k \) with a given \( \alpha \) = \min(2\epsilon_k, \sigma_{\tilde{g}}, \sigma_{\tilde{g}}) \) such that \( \delta_z > \frac{\beta}{\alpha} \) is satisfied. For example, \( \gamma_{\tilde{g}}, \gamma_{\tilde{g}}, \) and \( \mu_k \) can be set as follows:

\[ \begin{align*}
\mu_k &\leq \frac{\alpha \delta_z^2}{\epsilon_1 \mu D_k} \\
\gamma_{\tilde{g}} &\geq \frac{\epsilon_1 \sigma_{\tilde{g}} \sigma_{\tilde{g}}}{2\epsilon_1 D_k} \geq \frac{\epsilon_1 \sigma_{\tilde{g}} \sigma_{\tilde{g}} \sigma_{\tilde{g}}}{2\alpha \delta_z^2} \\
\gamma_{\tilde{g}} &\geq \frac{\epsilon_1 \sigma_{\tilde{g}} \sigma_{\tilde{g}} \sigma_{\tilde{g}}}{2\epsilon_1 D_k} \geq \frac{\epsilon_1 \mu_k \sigma_{\tilde{g}} \sigma_{\tilde{g}} \sigma_{\tilde{g}}}{2\alpha \delta_z^2}
\end{align*} \]

where \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) are any positive constants such that

\[ \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{1}{\epsilon_3} = 1 \]

**Remark 6.** Note that the choice of design parameters so as to satisfy \( \delta_z > \frac{\beta}{\alpha} \) depends on the size of the uncertainties. However as shown in Remark 1, by setting sufficiently small \( \mu_k \) and sufficiently large \( \gamma_{\tilde{g}} \) and \( \gamma_{\tilde{g}} \), it could be satisfied that \( \delta_z > \frac{\beta}{\alpha} \) even if one does not know enough \textit{a priori} information about uncertainties.

**V. NUMERICAL SIMULATION**

In this section, we confirm the effectiveness of the proposed robust adaptive control method by numerical simulation for the magnetic levitation ball system example given in [8]. The control objective is to keep the iron ball position \( y \) at a desired position \( y_r \). The typical model of the system is given by the following third-order state equations:

\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= g - \theta_1 \lambda(x_1) x_3^2 \\
\dot{x}_3 &= -\theta_2 x_1 + \theta_3 u
\end{align*} \] (55)

where \( x_1 = y, \ x_2 = \dot{y}, \ x_3 = I \) \( (I: \text{electromagnet current}) \) and \( u \) is the control voltage. \( g \) is the gravity and \( \theta_1 = 1/M \) \((M: \text{mass of the iron ball})\), \( \theta_2 = R/L, \) and \( \theta_3 = G/L \) are unknown parameters, where \( R, \ L, \) and \( G \) are the resistance, the inductance and the conductance, respectively, that regards the \( RL \)-relation between the electromagnet current \( I \) and the control voltage \( u \). Further, \( \lambda(x_1) \) is the unknown nonlinearity that regards the relationship between the force attracting the ball and the electromagnet current \( I \). In this simulation we assume that \( \theta_1 = 2, \theta_2 = 20, \theta_3 = 2 \) and

\[ \lambda(x_1) = \frac{150000}{2(0.25 + 900 x_1)^2} \] (56)

are unknown but the lower bound \( \theta_{\min} = 1 \) of \( \theta_3 \) and the function such that

\[ \lambda_{\min}(x_1) = \frac{80000}{(0.3 + 1000 x_1)^2} \leq \lambda_1(x_1) = 0, \lambda(x_1) \] (57)

are known.

The desired position \( y_r \) is set by

\[ y_r = 0.1 + G(s)[r], \quad G(s) = \frac{150}{(s+10)(s+15)} \] (58)

\[ r = \begin{cases} 
0 & \text{for } 0 \leq t < 3 \\
-0.02 & \text{for } 3 \leq t \leq 6
\end{cases} \]

It should be noted that the model is meaningful only for \( y > 0 \), and therefore the design will not yield a global result.

In order to apply the proposed method to the above stated levitation ball system, we introduce the new variable \( \tilde{x}_1 = x_2^2 \) so that the controlled system (87) can be represented by

\[ \begin{align*}
\dot{\tilde{x}}_1 &= \tilde{x}_2 \\
\dot{\tilde{x}}_2 &= g - \theta_1 \lambda(x_1) \tilde{x}_3 \\
\dot{\tilde{x}}_3 &= -\theta_2 \tilde{x}_1 + \theta_3 u
\end{align*} \] (59)

where \( \theta_2 = 20 \) and \( \theta_3 = 2 \).
For this system, we design the controller according to the proposed procedure by assuming that $f_1 = f_{10} = 0$, $g_1 = g_{10} = 1$ and $f_2 = f_{30} = g$ are known and

$g_2 = g_{20} = g_3 = -\theta_1 \lambda (x_1)$, $f_3 = f_{30} = \theta_2 x_3$,

$g_3 = g_{30} = \theta_3 x_3$

are unknown but

$g_{21} = -200$, $f_{31} = \bar{x}_3$, $g_{31} = x_3$

are known functions.

The design parameters are given in this simulation as follows:

$c_1 = 10, \quad c_2 = 10, \quad c_3 = 10$

$\gamma_{g2} = 100, \quad \gamma_{g3} = 10$

$\sigma_{g2} = 10, \quad \sigma_{g3} = 10$

$\delta_{\eta_{g21}} = 0.001, \quad \delta_{\eta_{g22}} = 0.05, \quad \delta_{\eta_{g31}} = 0.1$

$\delta_{\eta_{t2}} = 0.01, \quad \delta_{\eta_{t3}} = 0.001$

$\delta_2 = 0.0004, \quad \mu_1 = 1, \quad \mu_2 = 0.1, \quad \mu_3 = 0.01$

The initial values are set as

$x_1(0) = 0.05, \quad x_2(0) = 0, \quad x_3(0) = 0.1$

and

$\hat{\lambda}_{\text{final}}(0) = 100, \quad \hat{\theta}_2(0) = 0, \quad \hat{\theta}_3(0) = 1$

Figures 1 to 3 show the simulation results. As seen in Fig. 1, a good tracking performance is obtained. Figure 3 shows the tracking error signal $z_1$. We can see that the magnitude of the error signal $z_1$ is kept within the values of order $2 \times 10^2$ after about 0.5 second.

Figures 4 and 5 show the results using the ordinary adaptive backstepping strategy given in section 8.3 (Example: Levitation Ball) in [8] with a known nonlinear function $\lambda(x_1)$ and unknown parameters $\theta_1$ to $\theta_3$. In this simulation, we assumed that all the state variables are available for control so that the ordinary controller given in [8] is designed without a state observer. Thus the ordinary adaptive backstepping controller is designed as follows:

$z_1 = x_1 - y_r$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$

$\alpha_1 = -c_1 z_1 + \dot{y}_r$

$\alpha_2 = -\rho_1 \alpha_2$

$\alpha_{2b} = -c_2 z_2 - g + \beta_1 - z_1$

$\beta_1 = -c_1 x_2 + c_1 \dot{y}_r + \dot{y}_r$

The parameter adjusting laws are given by

$\mu = \frac{\hat{\rho}_2}{(2\lambda(x_1)x_3)} - \alpha_{3b}$

$\alpha_{3b} = -c_3 z_3 + 2\hat{\theta}_2 \lambda x_1 x_3 - \alpha_{2b} \hat{\theta}_1 - \hat{\theta}_1 \beta_2$

$+ \hat{\theta}_3 (\hat{\rho}_1 (c_1 + c_2) \lambda (x_1) x_3 + z_2) - \hat{\lambda}(x_1) \bar{x}_3 x_3$

$\beta_2 = -(c_1 + 1) x_2 - (c_1 + c_2) g + (1 + c_1 c_2) \dot{y}_r + c_1 \dot{y}_r + y_r^{(3)}$
The design parameters are set as follows in this simulation:

\[ c_1 = c_2 = 8, \quad c_3 = 4, \quad \gamma_{\rho_1} = \gamma_{\rho_2} = 1, \quad \gamma_{\theta_1} = \gamma_{\theta_2} = 10 \]

Figures 6 and 7 show the results in the case where there exists a mismatch of the nonlinear function in the controller such as

\[ \lambda(x_1) = \frac{100000}{2(0.3+1000x_1)^2} \]

If the nonlinear function \( \lambda(x_1) \) is known, then as seen in Fig. 4 one can obtain a good tracking performance. On the other hand, if there exists a mismatch in \( \lambda(x_1) \), then undesired performance appears as shown in Fig. 6.

VI. CONCLUSION

In this paper, we proposed a robust adaptive tracking control based on the backstepping strategy for strict-feedback nonlinear systems with non-parametric uncertain nonlinearities. It was shown that one can design a stable adaptive control system provided that the uncertain nonlinearities can be decomposed by an unknown bounded nonlinear function and a known nonlinear function. The proposed method can deal with uncertain nonlinearities that appear at the control input term. It was also shown that suitable choice of design parameters guarantees the convergence of tracking error to any desired bound independent of the largeness of the unknown bounded nonlinearities. The effectiveness of the proposed method was confirmed through a numerical simulation for a magnetic levitation ball system example.

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**APPENDIX A**

One can find functions $\tilde{\phi}_R(\eta_{R^*})$ and $\tilde{\phi}_N(\eta_{N^*})$, which make $\phi_\circ(\eta_\circ)$ and $\phi_\omega(\eta_\omega)$ into $C^{2n}$ and $C^{2n+1}$, in the following way for instance.

(i) Design example of $\tilde{\phi}_R(\eta_{R^*})$ for $\phi_\circ(\eta_\circ) \in C^{2n}$ and $C^{2n+1}$:

Set $\tilde{\phi}_R(\eta_{R^*})$ by

$$\tilde{\phi}_R(\eta_{R^*}) = \sum_{i=0}^n X_i \sin^{2i+1}\left(\frac{\pi}{2\delta_{R^*}}\eta_{R^*}\right)$$

(A.1)

Since $\tilde{\phi}_R(\eta_{R^*}) = 1$, $\tilde{\phi}_R(\eta_{R^*}) = 0$, ..., $\tilde{\phi}_R^{(2n)}(\eta_{R^*}) = 0$

(A.2)

we can find $X_i, i = 0, \ldots, n$.

(ii) Design example of $\tilde{\phi}_N(\eta_{N^*})$ for $\phi_\omega(\eta_\omega) \in C^{2n}$ and $C^{2n+1}$:

Set $\tilde{\phi}_N(\eta_{N^*})$ by

$$\tilde{\phi}_N(\eta_{N^*}) = \sum_{i=0}^n X_i \sin^{2i+1}\left(\frac{\pi(\eta_{N^*} + \delta_{N^*}/2)}{\delta_{N^*}}\right) - b$$

(A.3)

Since $\tilde{\phi}_N(\eta_{N^*}) = 0$, $\tilde{\phi}_N(\eta_{N^*}) = -1$, $\tilde{\phi}_N(\eta_{N^*}) = 0$, ..., $\tilde{\phi}_N^{(2n)}(\eta_{N^*}) = 0$

(A.4)

we can find $b$ and $X_i, i = 0, \ldots, n$.

**APPENDIX B**

For $R_1$, if $|\eta_{R^*}| > \delta_{R^*}$ we have from (5) and (7) that

$$R_1 \leq \tilde{f}_{10} |\eta_{R^*}| - \tilde{f}_{10} |\eta_{R^*}| + (\tilde{f}_{10} - \tilde{f}_{10}) |\eta_{R^*}| = 0$$

(B.1)

and in the case where $|\eta_{R^*}| \leq \delta_{R^*}$, we have from (9) that

$$R_1 \leq \tilde{f}_{10} |\eta_{R^*}| - \tilde{f}_{10} |\eta_{R^*}| + (\tilde{f}_{10} - \tilde{f}_{10}) |\eta_{R^*}| \leq \tilde{f}_{10} \delta_{R^*}$$

(B.2)
Therefore $R_{f1}$ can be evaluated by

$$R_{f1} \leq \bar{f}_{10} \delta_{Rg1}$$

Further for $R_{g1}$, if $|\eta_{Rg1}| > \delta_{Rg1}$ and $\eta_{ad1} > 0$, then we have from (6), (7), (8), and $\frac{\bar{f}}{G_{10}} > 0$ that

$$R_{g1} \leq \frac{\bar{g}_{10} - \frac{\hat{g}_{10}}{G_{10m}}}{} \eta_{1} \bar{z}_{1} - \frac{\bar{g}_{10} - \frac{\hat{g}_{10}}{G_{10}}}{} \eta_{i} \bar{z}_{i}$$

$$= 0 \quad (B.3)$$

if $|\eta_{Rg1}| > \delta_{Rg1}$ and $\eta_{ad1} < -\delta_{ad1}$ then we have

$$R_{g1} \leq |\eta_{1} \bar{z}_{1}| + 2 \frac{\bar{g}_{10}}{G_{10m}} |\eta_{i} \bar{z}_{i}| \leq 0 \quad (B.4)$$

moreover if $|\eta_{Rg1}| > \delta_{Rg1}$ and $-\delta_{ad1} \leq \eta_{ad1} \leq 0$ then we obtain

$$R_{g1} \leq \frac{\bar{g}_{10}}{G_{10}} \eta_{i} \bar{z}_{i} - \frac{\bar{g}_{10}}{G_{10m}} \eta_{i} \bar{z}_{i} + 2 \frac{\bar{g}_{10}}{G_{10m}} \eta_{1} \bar{z}_{1}$$

$$- \frac{\bar{g}_{10}}{G_{10}} |\eta_{1} \bar{z}_{1}| \leq 2 \delta_{ad1} \quad (B.5)$$

Furthermore, if $|\eta_{Rg1}| \leq \delta_{Rg1}$ and $\eta_{ad1} > 0$, then we have from $|\hat{\eta}_{Rg1}(\eta_{Rg1})| < 1$ that

$$R_{g1} \leq \frac{\bar{g}_{10}}{G_{10}} \eta_{i} \bar{z}_{i} - \eta_{1} \bar{z}_{i} + 2 \frac{\bar{g}_{10}}{G_{10m}} \eta_{1} \bar{z}_{1}$$

$$\leq \frac{\bar{g}_{10}}{G_{10}} \delta_{Rg1} \quad (B.6)$$

and if $|\eta_{Rg1}| \leq \delta_{Rg1}$ and $\eta_{ad1} < -\delta_{ad1}$, then we have

$$R_{g1} \leq -\eta_{1} \bar{z}_{1} + 2 \frac{\bar{g}_{10}}{G_{10m}} \eta_{1} \bar{z}_{1} + |\eta_{1} | \leq 0 \quad (B.7)$$

In the final case where $|\eta_{Rg1}| \leq \delta_{Rg1}$ and $-\delta_{ad1} \leq \eta_{ad1} \leq 0$, we obtain

$$R_{g1} \leq |\eta_{1} \bar{z}_{1}| - 2 \frac{\bar{g}_{10}}{G_{10m}} \hat{\eta}_{ad1} \eta_{1} \bar{z}_{1} + |\eta_{1} \bar{z}_{1}|$$

$$\leq 2 \delta_{ad1} \quad (B.8)$$

Finally $R_{g1}$ can be evaluated by

$$R_{g1} \leq \max(2 \delta_{ad1}, \frac{\bar{g}_{10}}{G_{10}} \delta_{Rg1})$$

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