HIERARCHICAL CONTROL SYSTEM FOR A VARIABLE SPEED CAGE MACHINE WIND GENERATION UNIT USING NEURAL NETWORKS

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ABSTRACT

A hierarchal control strategy, that addresses three control objectives for a wind generation system, is proposed in this paper. It controls the local bus voltage (to avoid voltage rise), captures the maximum power in the wind and also minimizes the power loss in the induction generator. In the first level, given the instantaneous wind speed, electrical torque and output power, the designed neural networks calculate the desired rotor speed, air-gap flux and the grid side reactive power. In the second level, the desired current wave shapes (instantaneous three-phase currents) of the rectifier and the inverter in a double-sided PWM converter system are calculated. In the third level, the PWM controller guides the system towards the optimum operating conditions. Simulation results show that even as the wind speed changes randomly, the proposed control strategy leads the system to the optimum operating conditions.

KeyWords: Wind energy, wind power generation, induction generators, modeling of power systems, voltage regulation.

I. INTRODUCTION

In response to increasing environmental concerns, more and more electricity is being generated from renewable sources. Harnessing of wind energy as a renewable source to generate electricity has developed extremely rapidly and many commercial wind turbines are now available on the market. The cost of generating electricity from wind has fallen almost 90% since the 1980s [1]. Worldwide installation of wind based generating capacity has exceeded that of nuclear based during recent years – an indication that wind is becoming a competitive player in today’s power market [2].

Wind is a variable and random source of energy. To convert this form of energy to electrical energy, all types of machines, i.e. DC, synchronous, induction, have been used; depending on the size of the system. Induction generators are more common and more economical in the range up to 2 MW [3].

Connecting an induction generator directly to a transmission line in a network without any control can be potentially troublesome and can result in both sub-optimal performance of the wind turbine and potential stability problems. The problems with such a simple connection are:

1. The magnitude of the voltage at the bus to which the unit is connected may rise to unacceptable levels [4,5]. This is because the existing transmission lines tend to be designed without any consideration for embedded generation.

2. Wind turbine output power vs. rotor speed is a nonlinear cubic function. A wind turbine can only deliver maximum possible power at a particular rotor speed. In a directly connected induction machine system the rotor speed is dictated by the electrical system frequency. Therefore, the maximum power of the wind is not normally captured.

3. The induction machine efficiency depends on the rotor speed and the rotor flux. The flux depends on the bus voltage and frequency in a directly connected induction machine system. Therefore, the maximum efficiency is
not normally achieved in the induction generator.

To have some level of control on the wind generation unit, various forms of systems can be used. In the simplest form, the wind generating unit is augmented by three-phase passive or active VAr compensators. Using this arrangement, only the terminal voltage can be controlled. Also, despite it being economical and reliable, a VAr compensating system severely limits the energy capture of the wind generating system [3].

The variable speed constant frequency systems have the advantage that the operators can control the rotor speed. This advantage makes it possible to capture maximum energy from the wind turbine. In the variable speed constant frequency systems, power electronic devices are used to allow the rotor speed to be changed while the grid frequency is constant.

If the rotor in the induction machine is a wound type rotor, power electronics can be used to change the frequency of the rotor current. Since in this configuration, the local bus feeds both the stator and rotor, the system is called a doubly fed wound rotor induction machine. Alternatively, a squirrel cage rotor induction machine can be used. When using a squirrel cage machine, the wind generation system is called a variable speed cage machine (VSCM) system. A VSCM system uses a rectifier and an inverter interposed between the cage induction generator stator and the grid [6-8].

A method of tracking the peak power points for a VSCM system is suggested in [9] and maximizing the penetration of wind generation in distribution networks is discussed in [10]. As the rectifier allows change of the rotor flux, the operator can also maximize the efficiency of the induction generator. In [11], fuzzy logic is used in a VSCM system to capture the maximum wind energy and maximize efficiency. The voltage rise problem has been addressed in [4] and some suggestions to overcome this problem are made. The effect of pitch control of the wind turbine of power quality has been addressed in [12] and sliding mode control strategy has been used for efficiency and torsional dynamics in [13].

In this paper a hierarchal control strategy is developed for a VSCM system in which all three control objectives, voltage rise, output power and efficiency, are taken into account, using only the rectifier and inverter control signals. Neural networks with Levenberg-Marquardt learning algorithm has been used to achieve optimum conditions despite random wind speed changes.

II. SCM WIND GENERATION SYSTEM

A simple block-diagram of a wind generation system is shown in Fig. 1. In this Figure, there are five main parts: wind profile, wind turbine, induction generator, power electronics (rectifier and inverter) and the external system (load and transmission line). The modeling of each section is discussed separately and then the overall model is investigated.

2.1 Wind profile

Wind speed changes continuously and its magnitude is random over any interval. To simulate the wind speed, it is common to assume that the mean value of the wind speed is constant for some intervals (for example every 10 minutes). The International Electro-technical Commission has recommended the use of Rayleigh probability distribution for the wind profile [14] to determine the ten-minute mean. To simulate a wind profile, sinusoidal fluctuations are usually added to the randomly changing mean value. A typical expression for the wind velocity, \( v \), is [14]:

\[
v = x (1 - 0.05 \cos(2\pi / 20) - 0.05 \cos(2\pi / 600))
\]

where \( x \) is the random number produced by Monte Carlo simulation. To simulate wind gusts, the magnitude and frequency of the sinusoidal fluctuations is increased.

For long-term studies (steady state), the sinusoidal fluctuations can be ignored, while for short-term studies (transients) the mean can be considered constant.

2.2 Wind turbine

The input of a wind turbine is the wind power (wind speed) and the output is the mechanical power turning the rotor. The output power from a wind turbine can be expressed as [3]:

\[
P_t = 0.5 \cdot C_p \cdot \rho \cdot A \cdot v^3
\]

where \( C_p \) is the power coefficient, \( v \) is the wind speed, \( \rho \) is the air density, \( A = \pi R^2 \) is the cross-sectional area of the turbine and \( R \) is the radius of the turbine. One disadvantage of (2) is that \( C_p \) depends on the turbine rotor speed, \( i.e. \ C_p = f(\omega) \). This relation is usually shown by a curve relating \( C_p \) to the tip-speed ratio (\( \lambda \)) defined by:

\[
\lambda = \frac{\omega R}{v}
\]
A typical $C_p$ vs. $\lambda$ curve is shown in Fig. 2. Wind turbine manufacturers usually provide this curve based on the characteristics of the turbine.

The power-speed characteristic of a wind turbine can be obtained using the $C_p$ vs. $\lambda$ curve and (2). A power versus speed curve of a wind turbine for various wind velocities with the following constant values $[3]$.

\[
R = 13.5\text{m}, A = 577\text{m}^2, \rho = 1.225\text{kg/m}^3, (n1 : n2) = (1 : 23)
\]

is shown in Fig. 3. To calculate this characteristic, the gear ratio $(n1 : n2)$ must also be taken into account.

### 2.3 Induction generator

An appropriate model of the induction generator is the most complicated part of the total wind generation model. The model of such a system is well described in many books and papers. Equations taken from $[15]$, used to form a fifth order model, are described in the Appendix.

### 2.4 Power system connection

To be able to simulate the induction generator and wind generation system, an equation relating $v_{ds}$, $v_{qs}$, the stator direct and quadrature axis voltages, to $i_{ds}$, $i_{qs}$, the stator direct and quadrature axis currents, is required. This relationship, required to use a current based power flow grid model, is obtained depending on the configuration of the terminal connection to the load. If the induction generator is connected to a constant voltage bus through a transmission line as well as local load, as shown in Fig. 1, then:

\[
I = Y \cdot V + (V - V_B)/Z
\]

where $Z$ is the transmission line impedance, $Y = (G + jB)$ is the local load admittance, $V$ is the terminal bus voltage, and $V_B$ is the infinite bus voltage. Since,

\[
I = i_{ds} + j i_{qs} \quad \text{and} \quad V = v_{ds} + jv_{qs}
\]

(3) can be re-written in terms of the d and q axis as,

\[
i_{ds} + j i_{qs} = (G + jB) \cdot (v_{ds} + jv_{qs})
\]

\[
+ (v_{ds} + jv_{qs} - v_{ds} - jv_{qs}) / (R + jX)
\]

giving the required mathematical relationship between the variables $v_{ds}$, $v_{qs}$ to $i_{ds}$, $i_{qs}$.

### 2.5 Power electronics

The double-sided pulse width modulator (PWM) converter system $[3]$, helps to reduce the harmonics in the wind generation system. PWM produces a train of pulses with variable width, such that the mean value (filtered signal) is proportional to the desired signal. Such power electronics systems are usually considered very fast. Therefore, their time constants, compared with the rotor and the induction machine dynamics are neglected in simulation studies. With this assumption, the rectifier and inverter can be modeled by nonlinear static functions. In order to perform an accurate simulation the following steps are taken:

1. Produce a triangular waveform with defined carrier frequency $\omega_c$, by:

\[
Tri(t) = \frac{2}{\pi} \sin^{-1}[\sin(\omega_c t)]
\]

2. Define a desired signal.

3. Compare the desired signal with a triangular wave to produce the train of pulses (e.g. the pulse value is 1 when the desired signal is greater than the triangular wave).

A plot of the PWM pulse train for one of the phases in a three-phase inverter is shown in Fig. 4.

If harmonics are not of interest, the rectifiers and inverters can easily be modeled by simple gains $[15]$. In the study presented in this paper, a current feedback and a gain
is used to model the current-controlled PWM rectifier and inverter for each phase, as the optimum operation and control is the main objective and the effect of harmonics is not of interest. More details are given in Section IV.

2.6 The overall wind generation system

To simulate the overall system, the mean wind speed is determined using Monte Carlo simulation, and sinusoidal fluctuations are added to generate instantaneous wind speed. For a given wind speed, the operating point of the wind turbine (mechanical output power and rotor speed) is determined by the intersection between the turbine characteristic and the load characteristic (induction generator). The external system is used to determine the stator voltages that are then used in the induction machine simulation.

In this paper, the machine equations are transformed using a rotor flux frame. With sinusoidal excitation, the rotor flux rotates with synchronous speed, but at a different angle than the stator flux. Selecting the $d$-axis aligned with the rotor flux, the $q$-axis component of the flux is zero. This makes the equations easier to handle. In this frame, the torque and flux equations described in the Appendix [15] can be rewritten as:

$$\Psi_r = x_r \cdot i_{qr} + x_m \cdot i_{qs} = 0 \Rightarrow i_{qs} = -\frac{x_m}{x_r} i_{qr}$$

$$T_e = -\frac{3P}{4} i_{qr} \cdot \lambda_{dr} = 3P \frac{x_m}{4} x_r i_{qr} \lambda_{dr}$$

$$\omega_e - \omega_r = \frac{R_s}{L_s} i_{qs}$$

$$\lambda_{dr} = \frac{R_s \cdot L_m}{R_s + L_r \cdot p} i_{ds}$$

Equations (4) are the basis for field oriented control [15,17]. This approach simplifies the induction machine control. The model is very similar to a separately excited DC machine where the flux depends on the field current and the torque is proportional to the flux and the armature current. The main problem associated with field oriented control is the requirement to estimate the flux axis angle. This is done either by measuring the flux at two different points (with 90 degrees displacement), or estimating through rotor speed measurement [15].

III. CONTROL OBJECTIVES IN A VSCM WIND GENERATION SYSTEM

The control objective in a VSCM wind generating system is to take into account all three control aspects, i.e., voltage rise, output power and efficiency. In this section, each aspect is described in detail.

The first control objective is to limit the voltage rise. Most distribution systems are designed to distribute power from large central power stations. Usually the range of voltage regulators does not extend beyond medium voltage bus bars. Connecting a wind generating unit to a bus bar with no voltage control will change the current flow in the buses and may result in a voltage rise and result in violation of limits. To safely use the wind generation unit, either a worst-case scenario is considered or an active voltage control is required [4,5]. Some of the strategies used in active voltage control are:

1. Reducing line impedance by changing the connection point.
2. Changing the active power of the local load (load control).
3. Changing the reactive power of the grid bus bar. This is mainly accomplished through the use of VAr compensators.
4. Changing the reactive power of the wind generator. This can be achieved by controlling the PWM converters in a VSCM configuration.

The second control objective is to capture maximum power from the wind. In Fig. 3, a typical power versus speed curve of a wind turbine is plotted. For a given wind speed, the mechanical output power of the wind turbine is maximum at a particular rotor speed. The best way to control the rotor speed is to change the frequency of the induction machine terminal voltage using the PWM rectifier. The current controlled PWM rectifier allows the change of terminal voltage frequency without affecting the system frequency.

The third control objective is to maximize the induction machine efficiency. The efficiency of an induction generator is a function of the rotor speed and the flux. As the optimum rotor speed at each instant is determined by the power speed characteristics of the wind turbine, the only way to improve the efficiency is to adjust the flux. During light load running conditions, the machine rotor flux can be reduced to reduce the core loss and therefore increase efficiency [15]. When the rotor flux is decreased,
the stator current should be increased to keep the torque or speed constant, but any increase in current results in higher copper loss.

Power loss (core loss plus copper loss) vs. flux characteristics at different speed (torque) levels for a 20Hp, 220V, 60Hz, 4-pole induction machine is shown in Fig. 5. Machine parameters are given in the Appendix. Given a particular torque (rotor speed) the optimum efficiency (minimum power loss) is obtained for a particular flux. Fig. 5 is obtained by first considering a range of variations for flux and torque. For each torque and flux, the direct and quadrature currents and therefore the magnitude of the current can be calculated using (4). The copper loss can be calculated by:

\[ P_{cu} = R_s |I_s|^2 \]

and the core loss can be approximated by:

\[ P_{core} = K_c |\phi|^2. \]

IV. CONTROL STRATEGY FOR A VSCM

In this section a hierarchal and comprehensive control strategy is proposed to achieve all the control objectives using the PWM rectifier and inverter based on the VSCM wind generating unit described in Section 2 and the control objectives described in Section 3. The general structure of the proposed controller is shown in Fig. 6.

There are three levels in the proposed controller:

**Level 1.** Given the control objectives (to capture the maximum wind power, to minimize the power loss in the induction generator and to control the local bus voltage at the desired value) and some variables of the wind generating system, calculate the desired rotor speed \( \omega_m^* \), the desired flux \( \phi_m^* \) and the desired reactive power delivered to or consumed from the grid \( Q^* \).

**Level 2.** Given the desired rotor speed, the desired flux, the desired reactive power and machine parameters, calculate the instantaneous input currents to the rectifier and the instantaneous output currents from the inverter for all three phases.

**Level 3.** Given the desired and measured instantaneous input currents to the rectifier and the desired and measured instantaneous output currents from the inverter, control the PWM converter to guide the system to the optimum conditions.

A detailed description of each level is given below:

**Level 1.**

The first step is to determine the desired rotor speed \( \omega_m^* \). This can be obtained by measuring the wind speed and determining \( \omega_m^* \) at which the mechanical output power is maximum, using the power speed characteristics of the wind turbine (for example, Fig. 3). To avoid time-consuming look-up tables, a neural network has been trained and used to produce the desired rotor speed, given the wind speed. The use of neural networks in this manner differs from traditional approaches to solve similar problems.

Once the desired rotor speed is calculated, the desired flux in the induction machine can be determined to achieve maximum efficiency. This can be achieved by measuring the rotor speed (or electrical torque) and determining \( \phi_m^* \) at which the power loss is minimum, using the power loss vs. flux characteristics of the induction generator (for example, Fig. 5). To avoid time-consuming look-up tables, a neural network can be designed to produce the desired flux, given the rotor speed.

Adjusting the reactive power of the inverter can be used to control the voltage rise of the grid side bus bar. Knowing the generated active power and desired voltage, the local grid side bus bar can be treated as a PV bus in a load flow analysis to calculate the desired reactive power \( Q^* \). Transmission line parameters, local load variations

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![Fig. 5. Power loss vs. flux at different rotor speeds (torques).](image-url)

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![Fig. 6. General structure of the controller.](image-url)
and other factors such as additional generators considered in the external system will influence the result of the controller. To avoid time-consuming load flow analysis, a neural network has been designed to determine the desired reactive power, given the active power and desired voltage.

The three neural networks can be trained online to follow any system changes.

**Level 2.**

In this section, the desired instantaneous input currents to the rectifier \(i_{\text{rec}}, i_{\text{rec}}, i_{\text{rec}}\) and the desired instantaneous output currents from the inverter \(i_{\text{inv}}, i_{\text{inv}}, i_{\text{inv}}\) are calculated using \(\omega^*_m, \varphi^*_m,\) and \(Q^*\).

The best way to determine \(i_{\text{rec}}, i_{\text{rec}}, i_{\text{rec}}\) from \(\omega^*_m\) and \(\varphi^*_m\) is through direct field control. In direct field control, the three-phase values are transformed to the rotating rotor flux axis. Using this transformation, the flux depends only on the direct current \(i_d\) and the torque depends only on \(i_q\) (and the flux). Therefore, (4) can be used to convert the desired \(\omega^*_m, \varphi^*_m\) to the desired \(i_{\text{rec}}, i_{\text{rec}}\). Using the rotor flux angle and the reference frame transformation, these values can be converted to \(i_{\text{rec}}, i_{\text{rec}}, i_{\text{rec}}\). Details of the appropriate reference frame transformation for direct field control are well documented in the literature, for example [14-17].

The inverter should pass the generated active power \(P^*\) and also match the desired reactive power \(Q^*\). At each instant, the induction machine output power \(P^*\) can be measured. Given \(P^*\) and \(Q^*\), \(i^*_{\text{dinv}}, i^*_{\text{qinv}}\) can be calculated. The two input signals \(i^*_{\text{dinv}}, i^*_{\text{qinv}}\) are then converted to a three-phase \(i_{\text{inv}}, i_{\text{inv}}, i_{\text{inv}}\) using Park’s transformation.

An alternative approach to calculate \(P^*\) is to measure the DC level of the output voltage of the rectifier and compare it with the desired value. The error then can be fed to a PI controller to produce \(P^*\).

**Level 3.**

In the rectifier, the desired instantaneous input currents for all three phases are compared with the measured values. The error is then used in the simple gain model or accurate model to calculate the instantaneous induction machine terminal voltage. In the current-controlled PWM inverter, however, the desired instantaneous output currents for all three phases are compared with the measured values. The error is then used to calculate the instantaneous local bus voltage. The local bus is connected to a constant voltage bus through a transmission line and local load.

**V. SIMULATION RESULTS**

In this case study, in level 1 of the controller, the wind speed, rotor speed and induction machine terminal active power are measured and used to calculate \(\omega^*_m, \varphi^*_m\), and \(Q^*\) in the three neural networks. The three neural networks were designed as feed forward neural nets with 5 neurons in the hidden layer. The tan-sigmoid transfer function was chosen to generate the outputs in the hidden layer.

The Levenberg-Marquardt learning algorithm [18] was used as the training algorithm for the numerical simulations. The Levenberg-Marquardt algorithm uses an approximation to the Hessian matrix in the following Newton-like update:

\[
W_{k+1} = W_k + (J^TJ + \mu I)^{-1}J^te
\]

where \(W\) is a vector containing all the weights in the neural network, \(e\) is the error vector of the desired output and the outputs of the network, \(J\) is the Jacobian matrix of the error vector with respect to the weights and \(\mu\) is a pre-selected value between zero and one. When the scalar \(\mu\) is zero, the algorithm is simply the Newton’s method, using an approximate Hessian matrix. When \(\mu\) is large, this becomes gradient descent with a small step size [18].

To show the performance of the proposed controller different simulation results with the proposed controller are presented. The desired rotor speed, flux and reactive power were then used in level 2 of the controller to calculate the desired rectifier and inverter instantaneous currents. In level three the induction machine terminal voltage and grid side voltage are determined.

**5.1 Variable wind speed**

In this study, the magnitude of the wind fluctuations was considered to be 5% with periods of 10 and 60 s. The instantaneous current in one of the phases \(i(t)\), relative rotor speed \(\alpha_i/\omega_m\), electrical torque \(Te\), active power of the induction machine \(Pe\), terminal voltage of the induction machine \(Vt\), grid side bus voltage \(Vb\) and grid side active power \(Pe\) are shown in Fig. 7. Despite the variation of wind and consequently variation of power and voltage in the induction machine, the grid side bus voltage is well regulated. A plot of the maximum attainable power from instantaneous wind speed and the actual output power of the wind generation system is shown in Fig. 8. It is seen that the output power closely follows the maximum attainable power. A plot of the minimum attainable power loss and the actual power loss of the system is shown in Fig. 9. This figure also demonstrates that the control system is satisfactorily changing the field flux to obtain maximum efficiency (or minimum power loss). In Figs. 8 and 9 the error is very small, such that the two are indistinguishable. The reason is that the dynamics of the wind (and therefore power) fluctuations are much slower than dynamics of the system. To show the difference a step change in the wind speed was applied.

**5.2 Step change in wind speed**

The same results as in Figs. 7, 8, and 9 but following a
step change in wind speed are shown in Figs. 10, 11, and 12. Although the induction machine voltage changes, the grid side voltage is well regulated by the inverter. In these figures the dynamics of the system dictate some errors between optimum performance (maximum efficiency and minimum power loss) and the actual values during transients.

5.3 Wind gust

Figure 13 shows the results when the wind speed undergoes gusty conditions. In this study, the magnitude of fluctuation was considered to be 25% and the period was considered to be 1 s. The results show that the system is capable of remaining stable and work at optimum conditions.
VI. CONCLUSION

A nonlinear model for complete variable speed cage induction machine (VSCM) wind-generating unit has been developed. In the model wind profile, wind turbine, induction generator, rectifier, inverter, local load, transmission line and infinite bus have been considered. A hierarchal and comprehensive control strategy for the wind generation system is suggested and developed. The control strategy that incorporates neural networks, leads the wind generation system to capture the maximum power from the wind and make the induction machine work with higher efficiency by changing the flux in the air-gap. It also controls the terminal voltage. All the control objectives are achieved through a double-sided PWM converter. Simulation studies show that the proposed control strategy effectively leads the system to an optimum point. A supplementary controller can be used to improve system stability during transients. Such a controller is under investigation.

REFERENCES

APPENDIX

Since in an induction machine, the mutual inductances of the rotor and stator change with rotor position and time, Park’s transformation is usually used to overcome the complexity of the model. If such a transformation is used, the final equations are [15]:

\[ v_{ds} = R_s i_{ds} + \frac{1}{\omega_b} \frac{d}{dt} \psi_{qs} - \omega_b \psi_{qs} \]

\[ v_{qs} = R_s i_{qs} + \frac{1}{\omega_b} \frac{d}{dt} \psi_{ds} + \omega_b \psi_{ds} \]

\[ v_b = R_s i_b + \frac{1}{\omega_b} \frac{d}{dt} \psi_{br} \]

\[ 0 = R_s i_d + \frac{1}{\omega_b} \frac{d}{dt} \psi_{dr} - (\omega_r - \omega_b) \psi_{qr} \]

\[ 0 = R_s i_q + \frac{1}{\omega_b} \frac{d}{dt} \psi_{dq} + (\omega_r - \omega_b) \psi_{qr} \]

\[ 0 = R_s i_b + \frac{1}{\omega_b} \frac{d}{dt} \psi_{br} \]

(A.1)

where \( v_{ds}, v_{qs}, v_b, i_{ds}, i_{qs}, i_b \) are the transformed terminal (stator) voltages and currents and \( v_{dr}, v_{dq}, v_{br}, i_{dr}, i_{dq}, i_{br} \) are transformed rotor voltages and currents and:

\[ \psi_{ds} = x_s \cdot i_{ds} + x_m \cdot i_{br} \]

\[ \psi_{qs} = x_s \cdot i_{qs} + x_m \cdot i_{br} \]

For a balanced load \( i_{ds} = i_{qs} = 0 \). Substituting for \( \psi \)'s, the equations become:

\[ \frac{1}{\omega_b} \frac{d}{dt} \psi_{ds} = v_{ds} - R_s i_{ds} + \frac{\omega_r}{\omega_b} \psi_{qs} \]

\[ \frac{1}{\omega_b} \frac{d}{dt} \psi_{qs} = v_{qs} - R_s i_{qs} - \frac{\omega_r}{\omega_b} \psi_{ds} \]

\[ \frac{1}{\omega_b} \frac{d}{dt} \psi_{dr} = R_s i_{dr} + \frac{(\omega_r - \omega_b)}{\omega_b} \psi_{qr} \]

\[ \frac{1}{\omega_b} \frac{d}{dt} \psi_{dq} = -R_s i_{dq} - \frac{(\omega_r - \omega_b)}{\omega_b} \psi_{qr} \]  

(A.2)

For simple simulation in SIMULINK, some manipulations of the above equations are required, to avoid algebraic loops. If

\[ \psi_{mq} = x_m \cdot (i_{ds} + i_{dq}) \]

\[ \psi_{md} = x_m \cdot (i_{ds} + i_{dq}) \]

then

\[ \psi_{qs} = x_b \cdot i_{ds} + \psi_{mq} \]

\[ \psi_{ds} = x_b \cdot i_{ds} + \psi_{md} \]

\[ \psi_{qs} = x_b \cdot i_{ds} + \psi_{mq} \]

\[ \psi_{qs} = x_b \cdot i_{ds} + \psi_{md} \]

where \( x_b = x_s - x_m \); \( x_q = x_r - x_m \). Also one can write:

\[ i_{ds} = \frac{\psi_{ds}}{x_b} \]

\[ i_{ds} = \frac{\psi_{ds} - \psi_{mq}}{x_b} \]

\[ i_{dq} = \frac{\psi_{dq} - \psi_{mq}}{x_q} \]

\[ i_{dq} = \frac{\psi_{dq} - \psi_{mq}}{x_q} \]

and

\[ \psi_{mq} = x_M \left( \frac{\psi_{qs}}{x_b} + \frac{\psi_{dq}}{x_q} \right) \]

\[ \psi_{md} = x_M \left( \frac{\psi_{ds}}{x_b} + \frac{\psi_{dq}}{x_q} \right) \]

where

\[ x_M = \frac{1}{\frac{1}{x_s} + \frac{1}{x_q} + \frac{1}{x_b}} \]

The other main equation of the induction generator is
the rotor dynamics. According to Newton’s law:

\[ T_m - T_r - T_d = J \frac{d\omega_r}{dt} \]

where \( T_m, T_r, T_d \) are mechanical, electrical and damping torques respectively. Usually \( T_d = D \cdot \omega_r \) is assumed. Therefore the dynamics of the rotor becomes:

\[ T_m - T_r = J \frac{d\omega_r}{dt} + D \cdot \omega_r \]

(A.4)

In [15], it is shown that:

\[ T_r = \frac{3}{4} P L_p (i_{d*} \cdot i_{q*} - i_{q*} \cdot i_{d*}) = \frac{3P}{4 \cdot \omega_b} (i_{q*} \cdot \psi_{ds} - i_{d*} \cdot \psi_{qs}) \]

where \( P \) is the number of poles in the induction generator. Equations (A-3) and (A-4) describe the dynamics of an induction generator.

Machine parameters considered in the simulation are:

- \( S_b = 20 \times 746; \) rating, VA
- \( V_{rated} = 220; \) rated line-to-line voltage, V
- \( p_f = 0.853; \) rated power factor
- \( P = 4; \) number of poles
- \( f = 60; \) rated frequency, Hz
- \( R_s = 0.1062; \) stator winding resistance, ohms
- \( x_s = 0.2145; \) stator leakage reactance, ohms
- \( x_o = 0.2145; \) rotor leakage reactance, ohms
- \( x_m = 5.8339; \) stator magnetizing reactance, ohms
- \( R_r = 0.0764; \) referred rotor winding resistance, ohms
- \( J = 2.8; \) rotor inertia, kg \( \cdot \) m^2
- \( D = 0; \) rotor damping coefficient
- \( K_c = 2000; \) core loss constant
- \( Z = R + jX = j1.5; \) transmission line impedance, ohms
- \( Y = (G + jB); \) local load admittance

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