FAULT DETECTION, ISOLATION AND RECONSTRUCTION FOR DESCRIPTOR SYSTEMS

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ABSTRACT

In this paper, we consider fault detection, isolation and reconstruction problem for descriptor systems with actuator faults and sensor faults, respectively. When actuator faults exist in the system, the fault detection and isolation (FDI) problem is solved through an unknown input observer regarding remaining faults excluded a specified fault as unknown inputs. Whereas, in existing sensor faults, the fault detection is only achieved by the unknown input observer and residual signals. Since the derivative signal of sensor fault is generated in the error dynamics between the actual system and the derived observer. The main objective of this work attempts the reconstruction of the faults. The reconstruction can be achieved by sliding mode observer including feedforward injection map and compensation signal. Finally, the isolation problem of sensor faults is solved by reconstructing all of the faults.

KeyWords: Fault detection, isolation and reconstruction, descriptor system, unknown input observer, sliding mode approach.

I. INTRODUCTION

The standard controller has been designed to maintain satisfactory operations by compensating for the effects of disturbance and changes occurring in the process. While these controllers can compensate for many types of disturbances, there are changes in the process which the controllers cannot handle adequately. These changes are called faults. More precisely, a fault is defined as an unpermitted deviation of at least one characteristic property or variable of the system [1]. The types of faults occurring in industrial systems include process parameters changes, disturbances parameter changes, actuator problems, and sensor problems [2]. Since these faults tend to degrade the overall system performance, they must be detected, isolated and be diagnosed.

The process of detecting and isolating those faults has been of considerable interest during the last two decades [3-14]. The research is still under way into the development of more effective solution for fault detection and isolation (FDI) in automatic control systems. The purpose of fault detection is to determine that a fault has occurred in the system, whereas fault isolation procedures are used to determine the location of the fault, after detection. The most obvious method for automatic fault detection is the use of hardware redundancy, where measurements from multiple sensors are compared with each other and the existence of a fault is determined by implementing a voting mechanism. In many situations, however, hardware redundancy may not be possible or desirable, since it imposes a penalty in terms of volume, weight and costs etc. In other situations such as with actuators, direct measurement is often not possible. In these cases, indirect measurements may be used to infer the component fault status using model of the system. On this FDI approach, the most effective way is the observer based approach in which the difference between actual and estimated outputs is used.
observers for descriptor systems. Some approaches are based on a singular value decomposition (SVD) [21,22] and lead to the construction of reduced-order or full-order observers. Other approaches utilize the concept of generalized matrix inverse [23] in order to construct under certain conditions minimal-order observers for this class of systems. For FDI problem of descriptor systems, the a few works as [22] have considered based on the observers, since the structural complexity and strong constraints in designing procedures. In [25], M. Hou considered the detection and isolation of faults. He proposed the numerical design procedures for FDI observer design based on the decomposition of matrix pencils. In this way, the observer is designed under the assumption which sensor faults can be represented as pseudo-actuator faults. However, the existence condition of decomposition matrices are very strict, the design is also difficult.

In this paper, we consider fault detection, isolation and reconstruction problem for descriptor systems with actuator faults, and with sensor faults, respectively. When actuator faults exist in the system, firstly the faults are split a specified fault and the remaining faults. And then, an unknown input observer is designed in order to decouple the remaining faults. Next, residual signals are determined from properly weighted output errors between measurements and estimated outputs. Whereas, in existing sensor faults, the derivative signals of sensor faults are generated in the error dynamics. Since the design of a matrix decoupling the derivative signals, the detection of sensor faults is only achieved in this process. First of all, the main objective of this paper attempts the reconstruction of each fault using sliding mode observer with feedforward signals [16]. In this way, the feedforward signals are to guarantee the stability of the state error dynamics and the existence of sliding mode. To minimize the error of the fault reconstruction, the enhanced sliding mode observer including the previously reconstructed results is proposed. Particularly, the isolation problem of the sensor faults can be solved by reconstructing the whole faults. Finally, the effectiveness of this approach is shown through the numerical simulation for a chemical mixing tank system with standpipe.

Throughout this paper, the notation |·| and ||·|| indicate the euclidean norm for vectors and spectral norm for matrices, respectively and null (#) does null space of the matrix. Also, the superscript + does the generalized matrix inverse.

II. FDI PROBLEM FOR DESCRIPTOR SYSTEM

2.1 In existing actuator faults

Consider the following descriptor system with actuator faults
\[
\begin{align*}
E \dot{x}_a(t) &= Ax_a(t) + Bu(t) + \eta_a f_a(t) \\
y_a(t) &= Cx_a(t)
\end{align*}
\] (1)

where \(x_a(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^m\), \(f_a(t) \in \mathbb{R}^q\), and \(y_a(t) \in \mathbb{R}^p\) denote the state variables, the control input, the actuator fault and the output, respectively. The matrices \(E, A, B, C\), and \(\eta_a\) are constant and real-valued with appropriate dimensions. \(E\) is a singular matrix with rank \(E = r \leq n\). We assume the following

(i) The system (1) is regular, and observable in sense of Rosenbrock [22].
(ii) \(\eta_a\) is a full column rank,
(iii) rank \((C \eta_a) = q_a\), \((p \geq q_a)\).

To determine the existence of fault at \(i\)-th actuator, the system (1) may be described as

\[
\begin{align*}
E \dot{x}_a(t) &= Ax_a(t) + Bu(t) + \eta_a f_a^i(t) + \eta_a^f f_a^a(t) \\
y_a(t) &= Cx_a(t)
\end{align*}
\] (2)

where \(f_a^i(t) \in \mathbb{R}^q\) is \(i\)-th fault and \(f_a^a(t) \in \mathbb{R}^q\) is the remaining faults excepting for \(f_a^i(t)\). And it is assumed that function \(f_a^i(t)\) is bounded, i.e., there exists a non-negative real number \(m_a^i\) such that \(\|f_a^i(t)\|_\infty \leq m_a^i\).

Now, an unknown input observer and residual signals are designed in order to detect and isolate actuator faults.

For system (2), the unknown input observer is defined as

\[
\begin{align*}
\dot{\hat{z}}_a^i(t) &= N_a^i \hat{z}_a^i(t) + L_a^i y_a(t) + J_a^i u(t) \\
\dot{\hat{z}}_a^a(t) &= z_a^a(t) + V_a^i y_a(t)
\end{align*}
\] (3)

where \(\dot{\hat{z}}_a^i(t) \in \mathbb{R}^n\) and \(\hat{z}_a^i(t) \in \mathbb{R}^n\) are the estimated and transformed state vector, respectively. \(N_a^i, L_a^i, J_a^i, \) and \(V_a^i\) are unknown matrices with appropriate dimensions.

Let’s define two matrices, \(E_a^a \in \mathbb{R}^{n \times n}\) and \(C_a^a \in \mathbb{R}^{n \times p}\) such that

\[
\begin{bmatrix}
E_a^a \\
C_a^a
\end{bmatrix}
\begin{bmatrix}
E \\
C
\end{bmatrix} = I_n
\] (4)

where \(\begin{bmatrix}
E_a^a \\
C_a^a
\end{bmatrix}\) is generalized matrix inverse of \(\begin{bmatrix}
E^T \\
C^T
\end{bmatrix}^T\) [18] and \(E_a^a\) is non-singular matrix. Then, we can obtain the following lemma which describes the relation between system (2) and the unknown input observer (3) as such as [22].

**Lemma 1.** The system (3) is an unknown input observer for the descriptor system (2) if

\[
\lambda(N_a^i) < 0, \quad i = 1, 2, \ldots, q_a
\] (5)

and there exist the matrices \(R_a^i \in \mathbb{R}^{n \times m}\) and \(V_a^i \in \mathbb{R}^{p \times p}\) such that

\[
\begin{align*}
N_a^i R_a^i E + L_a^i C &= R_a^i A \\
J_a^i &= R_a^i B \\
R_a^i \gamma_a^i &= 0 \\
R_a^i E + V_a^i C &= I_n
\end{align*}
\] (6)-(9)

where \(\text{Re} \lambda(\cdot)\) denotes the real part of eigenvalues.

**Proof.** The estimation error \(\xi_a^i(t)\) is defined as

\[
\xi_a^i(t) = \hat{z}_a^i(t) - R_a^i E x_a(t)
\] (10)

then, the estimation error dynamics is governed as

\[
\dot{\xi}_a^i(t) = N_a^i \xi_a^i(t) + (N_a^i R_a^i E + L_a^i C - R_a^i A) x_a(t) + (J_a^i - R_a^i B) u(t) - R_a^i \gamma_a^i f_a^i(t)
\] (11)

by invoking (5)-(9) which is reduced as

\[
\dot{\xi}_a^i(t) = N_a^i \xi_a^i(t) - R_a^i \gamma_a^i f_a^i(t)
\] (12)

The remaining problem is to determine matrices \(R_a^i\) and \(V_a^i\) to satisfy the above lemma. From (9), we can get

\[
R_a^i E = I_n - V_a^i C
\] (13)

Substitution of (4) into (13) yields

\[
R_a^i E = R_a^i (E_a^a)^{-1} (I_n - C_a^a C) = I_n - V_a^i C
\] (14)

from which the matrix \(R_a^i\) is calculated as

\[
R_a^i = (I_n - M_a^i C) E_a^a
\] (15)

where

\[
M_a^i = V_a^i - R_a^i (E_a^a)^{-1} C_a^a
\]

Substitution of (15) into (14) yields

\[
R_a^i E = (I_n - M_a^i C)(I_n - C_a^a C)
\] (16)

Since rank \(C = p\), we find

\[
V_a^i = C_a^a + M_a^i (I_p - C_a^a C)
\] (17)

By substituting (13) into (6), we have

\[
N_a^i = R_a^i A - K_a^i C
\] (18)
where
\[ K_a = L_a - N_a V_a' \]

Also, by using (15) and (18), the matrix \( E_a \) is

\[ L_a = N_a V_a' + K_a = K_a (I_p - CV_a') + (I_a - M_a C) E_a A V_a' \quad (19) \]

Next, from (8) and (15), we obtain

\[ M_a C E_a^\flat \eta_a^\flat = E_a^\flat \eta_a^\flat \quad (20) \]

whose solution exists if

\[ \text{rank}(C E_a^\flat \eta_a^\flat) = \text{rank} \eta_a^\flat = q_a^\flat \]

which is proved from the previous assumption and the non-singular matrix \( E_a^\flat \). The general solution of (20) can be written as

\[ M_a = E_a^\flat \eta_a^\flat (C E_a^\flat \eta_a^\flat)^\flat + Q_a^\flat (I_a - E_a^\flat \eta_a^\flat (C E_a^\flat \eta_a^\flat)^\flat) \quad (21) \]

where \( Q_a^\flat \) is arbitrary matrix.

By substituting \( M_a \) into (15), we yield

\[ R_a = (I_a - Q_a^\flat C) (I_a - E_a^\flat \eta_a^\flat (C E_a^\flat \eta_a^\flat)^\flat C) E_a^\flat \quad (22) \]

where the matrix \( K_a \) is a general feedback gain stabilizing the matrix \( N_a \) under the condition which \( (R_a, A, C) \) is observable.

**Remark 1.** For the descriptor system (2), an unknown input observer (3) exists if

(i) \( \text{rank}(CE_a^\flat \eta_a^\flat) = \text{rank} \eta_a^\flat = q_a^\flat \),

(ii) \( \text{rank} \begin{bmatrix} A - sE & \eta_a^\flat \\ C & 0 \end{bmatrix} = n + q_a^\flat, \quad \forall s \in \mathbb{C}, \quad \Re(s) \geq 0 \)

where \( \mathbb{C} \) denotes complex plane.

From Lemma 1, the unknown input observer was designed. Now, the problem is to check whether \( f_a(t) \) exists or not. In order to decide the existence, the method using a residual signal is introduced.

A state error, \( w_a^F(t) = \hat{x}_a^F(t) - x_a^F(t) \) is defined. From (3) and (10), the state error may be described as

\[ w_a^F(t) = \hat{x}_a^F(t), \quad u_a^F(t) = \hat{x}_a^F(t) \]

The residual signal \( r_a^F(t) \) is defined as

\[ r_a^F(t) = C \xi_a^F(t) \quad (23) \]

And the simple threshold logic is determined as

\[ \| r_a^F(t) \| < \text{threshold for no fault cases} \]

\[ \| r_a^F(t) \| \geq \text{threshold for fault cases} \]

Finally, the actuator fault \( f_a(t) \) is detected and isolated from the above logic and the observer (3).

### 2.2 In existing sensor faults

Consider the descriptor system with sensor faults as follows

\[ \begin{align*}
E_a \dot{x}_a(t) &= A x_a(t) + Bu(t) \\
y_a(t) &= C x_a(t) + f_a(t)
\end{align*} \quad (24) \]

where \( f_a(t) \in \mathbb{R}^{q_a} \) is the sensor fault. We assume the following

(i) \( \eta_a \) is full column rank,

(ii) The number of measurement is greater than sensor faults, i.e., \( p > q_a \).

For system (24), an unknown input observer may be described as

\[ \begin{align*}
\dot{z}_a(t) &= N_a z_a(t) + L_a y_a(t) + J_a u(t) \\
\dot{\chi}_a(t) &= z_a(t) + V_a y_a(t)
\end{align*} \quad (25) \]

where \( \dot{\chi}_a(t) \in \mathbb{R}^n \) and \( z_a(t) \in \mathbb{R}^n \) are the estimated and transformed state vector, respectively. \( N_a, L_a, J_a, V_a \) are unknown matrices of appropriate dimensions.

Let's define two matrices, \( E_a^\flat \in \mathbb{R}^{n \times n} \) and \( C_a^\flat \in \mathbb{R}^{n \times p} \) such that

\[ \begin{bmatrix} E_a^\flat \\ C_a^\flat \end{bmatrix} = I_n \quad (26) \]

where rank \( C_a^\flat = p \) must be satisfied.

**Lemma 2.** The system (25) is an unknown input observer for system (24) if

\[ \Re \lambda(N_a) < 0 \quad (27) \]

and there exist matrices \( R_a \in \mathbb{R}^{n \times n} \) and \( V_a \in \mathbb{R}^{n \times p} \) such that

\[ \begin{align*}
N_a R_a E + L_a C &= R_a A \\
J_a &= R_a B \\
R_a E + V_a C &= I_n
\end{align*} \quad (28)-(30) \]

**Proof.** The proof is omitted. \[ \blacksquare \]
An estimation error $\xi_s(t)$ is defined as

$$\xi_s(t) = z_s(t) - R_s E x_s(t)$$  \hspace{1cm} (31)

From (24) and (25), estimation error dynamics is obtained as

$$\dot{\xi}_s(t) = N_s \xi_s(t) + (N_s R_s E + L_s C - R_s A) x_s(t) + (J_s - R_s B) u(t) + L_s \eta_s f_s(t)$$  \hspace{1cm} (32)

which by invoking (28) and (29) reduces to

$$\dot{\xi}_s(t) = N_s \xi_s(t) - L_s \eta_s f_s(t)$$  \hspace{1cm} (33)

If state error is $\hat{x}_s = z_s - x_s$, the error dynamics can be described from (25), (31), and (33) as

$$\dot{w}_s(t) = \dot{\xi}_s(t) - V_s \eta_s \dot{f}_s(t)$$  \hspace{1cm} (34)

where we can see that derivative signal, $V_s \eta_s \dot{f}_s(t)$ is to generate. To detect sensor faults, the derivative signals must be handled.

**Theorem.** Sensor faults are detected by decoupling the derivative signals, if $V_s \eta_s = 0$  \hspace{1cm} (35)

**Proof.** Through the similar process as Lemma 1, $V_s$ is obtained as

$$V_s = (I_s - G_s E) C^s$$  \hspace{1cm} (36)

and $R_s$ is written as

$$R_s = E^s + G_s (I_s - E E^s)$$  \hspace{1cm} (37)

From (35) and (36), we get

$$G_s E^s \eta_s = C^s \eta_s$$  \hspace{1cm} (38)

The general solution of (38) can be written as

$$G_s = C^s \eta_s (EC^s \eta_s)^* + Q (I_s - EC^s \eta_s (EC^s \eta_s)^*)^*$$  \hspace{1cm} (39)

where $Q$ is an arbitrary matrix. By substituting (39) into (36), we can get

$$V_s = (I_s - Q_s E) (I_s - C^s \eta_s (EC^s \eta_s)^*) C^s$$  \hspace{1cm} (40)

By substituting $G_s$ into (37), the matrix $R_s$ is obtained. $\blacksquare$

The error dynamics between (24) and (25) is given by

$$\dot{w}_s(t) = \dot{\xi}_s(t) = N_s \xi_s(t) - L_s \eta_s f_s(t)$$

As (23), the residual signal for sensor faults is defined as $r_s(t) = C \xi_s(t)$  \hspace{1cm} (41)

from which, we can see whether sensor faults are generating or not, but can not see where the faults exist. That is, the isolation of sensor faults is not achieved from (41).

**Remark 2.** For the descriptor system with sensor fault (24), the unknown input observer (25) exists if

(i) $\text{rank}(EC^s \eta_s) = \text{rank} \eta_s = q_s$

(ii) $\text{rank} \begin{bmatrix} A - sE & 0 \\ C & \eta_s \end{bmatrix} = n + q_s, \forall s \in \mathbb{C}, \text{Re}(s) \geq 0$

3. FAULT RECONSTRUCTION USING SLIDING MODE OBSERVER

In practice systems, to estimate the shape and severity of generating faults are very important. A.J. Koshkouei [14] has tried the estimation of actuator faults for general linear systems. However, in most previous works, FDI problems have been considered by residual signals. In this section, the fault reconstruction is achieved using sliding mode observer including feedforward signals.

3.1 Reconstruction of actuator faults

Let’s assume $i$-th actuator fault was detected through the process of Section 2. As adding a discontinuous input to obtained unknown input observer, the sliding mode observer may be determined as

$$\begin{align*}
\dot{z}_{i\omega}(t) &= N_s z_{i\omega}(t) + L_s y_s(t) + J_s u(t) \\
\dot{\xi}_s(t) &= z_{i\omega}(t) + V_s \eta_s \dot{f}_s(t)
\end{align*}$$  \hspace{1cm} (42)

where $V_s \eta_s \dot{f}_s(t)$ is an external feedforward compensation signal and $\lambda_s \in \mathbb{R}^{n \times p}$ is the feedforward injection map, and $(N_s, \lambda_s)$ is completely controllable and $C \lambda_s \neq 0$.

Subtracting (2) from (42), the error dynamics is

$$\begin{align*}
\dot{w}_s(t) &= \dot{\xi}_s(t) - V_s \eta_s \dot{f}_s(t) \\
\dot{y}_s(t) &= y_s(t) - C \xi_s(t)
\end{align*}$$  \hspace{1cm} (43)

where $\lambda_s \neq 0$ is an external feedforward compensation signal and $\lambda_s \in \mathbb{R}^{m \times p}$ is the feedforward injection map, and $(N_s, \lambda_s)$ is completely controllable and $C \lambda_s \neq 0$.

Subtracting (2) from (42), the error dynamics is

$$\begin{align*}
\dot{e}_s(t) &= (N_s \dot{e}_s(t) + R_s \eta_s \dot{f}_s(t) - \lambda_s \dot{v}_s(t) \\
\dot{\xi}_s(t) &= z_{i\omega}(t) + V_s \eta_s \dot{f}_s(t)
\end{align*}$$  \hspace{1cm} (44)

The equation of ideal sliding mode may be obtained from conditions as

$$\dot{e}_s(t) = 0, \quad \dot{\xi}_s(t) = 0$$  \hspace{1cm} (45)

From (44) and (45), a virtual equivalent feedforward signal may be described as

$$e_\omega(t) = C \dot{\xi}_s(t)$$  \hspace{1cm} (46)
The discontinuous input \( v_j^d(t) \) is defined as

\[
v_j^d(t) = W_d^j \text{sgn}(e_j^i(t)) = \frac{e_j^i(t)}{\| e_j^i(t) \|} \cdot e_j^i(t) \neq 0 \tag{47}
\]

where \( W_d^j \) is a diagonal matrix with elements \( w_{a_j}^d \) as

\[
w_{a_j}^d = \frac{|C_k R_{q_j}^i \lambda_{i_j}^d|}{\| C_k \lambda_{i_j}^d \|}, \quad k = 1, \ldots, p \tag{48}
\]

where \( C_k \) denotes the \( k \)-th column of \( C \) and \( \lambda_{i_j}^d \) is the \( k \)-th row of \( \lambda_{i_j}^d \) respectively. Now, we establish a condition so that \( \lim \epsilon_j^i(t) = 0 \).

Let \( P_a^d \) be the solution of the Lyapunov equation as

\[
N_a^i P_a^d + P_a^d N_a^i = -Q_a^i \tag{49}
\]

where \( Q_a^i \) is an arbitrary p.d.s. matrix. A Lyapunov function candidate for (43) is

\[
V(\epsilon_j^i(t)) = \frac{\epsilon_j^i(t)^T P_a^d \epsilon_j^i(t)}{} \tag{50}
\]

where \( P_a^d \) is defined in (49). To lead the stability of error dynamics (43) and the existence of the sliding mode, the feedforward injection map \( \lambda_{i_j}^d \) is chosen as

\[
\lambda_{i_j}^d = P_a^{-1} C_r^T \tag{51}
\]

Then the following is considered

\[
R_{q_j}^i \eta_a^d = \lambda_{i_j}^d \rho_a^d + H_{t_k}^i \tag{52}
\]

where \( \rho_a^d \in \mathbb{R}^{n \times 1} \) and \( H_{t_k}^i \in \mathbb{R}^{n \times 1} \) are a constant matrix. To check the stability of error dynamics (43), the derivative of \( V(\epsilon_j^i(t)) \) is as follows

\[
\dot{V}(\epsilon_j^i(t)) = \epsilon_j^i(t)(N_a^i P_a^d + P_a^d N_a^i) \epsilon_j^i(t)  \\
+ 2 \epsilon_j^i(t)^T P_a^d R_{q_j}^i f_j^i(t) - 2 \epsilon_j^i(t) P_a^d \lambda_{i_j}^d v_j^i(t)  \\
\leq - \epsilon_j^i(t)^T Q_a^i \epsilon_j^i(t) + 2 \| \epsilon_j^i(t)^T P_a^d R_{q_j}^i \eta_a^d \| \| m_a^d \|  \\
-2 \epsilon_j^i(t)^T P_a^d \lambda_{i_j}^d W_d^j \left \| \frac{C_k R_{q_j}^i \lambda_{i_j}^d}{\| C_k \lambda_{i_j}^d \|} \right \|  \\
= - \epsilon_j^i(t)^T Q_a^i \epsilon_j^i(t) + 2 \| \epsilon_j^i(t)^T P_a^d R_{q_j}^i \eta_a^d \| \| m_a^d \|  \\
-2 \epsilon_j^i(t)^T P_a^d \lambda_{i_j}^d W_d^j \left \| \frac{C_k R_{q_j}^i \lambda_{i_j}^d}{\| C_k \lambda_{i_j}^d \|} \right \|  \\
\leq - \epsilon_j^i(t)^T Q_a^i \epsilon_j^i(t) + 2 \| \epsilon_j^i(t)^T P_a^d R_{q_j}^i \eta_a^d \| \| m_a^d \|  \\
-2 \| \epsilon_j^i(t)^T P_a^d \lambda_{i_j}^d W_d^j \| \tag{53}
\]

Now, we desire to find the existence of sliding mode. For this, the following must be satisfied for each output error.

\[
e_i^a(t) \text{sgn}(e_i^a(t)) = C_k N_a^i e_i^a(t) \text{sgn}(e_i^a(t))  \\
+ C_k (R_{q_j}^i \eta_a^d f_j^i(t) - \lambda_{i_j}^d v_j^i(t)) \text{sgn}(e_i^a(t))  \\
\leq C_k N_a^i e_i^a(t) \text{sgn}(e_i^a(t)) + |C_k R_{q_j}^i \eta_a^d| \| m_a^d \|- |C_k \lambda_{i_j}^d w_a^d|  \\
\leq |C_k N_a^i e_i^a(t)| + |C_k R_{q_j}^i \eta_a^d| \| m_a^d \|- |C_k \lambda_{i_j}^d w_a^d|  \\
\leq |C_k N_a^i e_i^a(t)| + |C_k R_{q_j}^i \eta_a^d| \| m_a^d \|- |C_k \lambda_{i_j}^d w_a^d| < 0 \tag{54}
\]

where \( e_i^a(t) \) denotes the \( k \)-th output error signal. From last inequality of (54), the following is obtained.

\[
\| e_i^a(t) \| < \frac{|C_k \lambda_{i_j}^d w_a^d| - |C_k R_{q_j}^i \eta_a^d| \| m_a^d \|}{|C_k N_a^i|} \tag{55}
\]

Note that (55) is the sufficient condition to exist sliding mode in the neighborhood of \( e_i^a(t) = 0 \). The right term in (55) is described a boundary, which the errors are able to exist on the sliding surface.

Under the stability of error dynamics and the existence of sliding mode, from (44) we get

\[
C_k \lambda_{i_j}^d v_j^i(t) = CR_{q_j}^i \eta_a^d f_j^i(t) \tag{56}
\]

To recover the equivalent feedforward signal, the discontinuous signal of (47) is replaced by the continuous signal as

\[
v_j^d(t) = W_d^j \frac{e_j^i(t)}{\| e_j^i(t) \|} + \delta_a^j \tag{57}
\]

where \( \delta_a^j \) is a small positive scalar. Then, the equivalent output injection can be approximated to any degree of accuracy by (57) for a small enough choice of \( \delta_a^j \). From (56) and (57), it follows that

\[
f_j^i(t) \approx (C R_{q_j}^i \eta_a^d)^T (C k \lambda_{i_j}^d) v_j^d(t) \tag{58}
\]

from which the reconstruction of actuator fault is achieved.
The reconstruction procedure can show as Fig. 1.

**Remark 3.** For \( f_a(t) \) in (2), the actuator fault can be reconstructed if

(i) \( C \lambda_a \neq 0 \)

(ii) \( \text{rank}(CR^T_a \eta_a) = \text{rank} \eta_a \)

**3.2 Reconstruction of sensor faults**

Let’s consider the following sliding mode observer for system (24).

\[
\begin{align*}
\dot{z}_s(t) &= N_s z_s(t) + L_s y_s(t) + J_s u(t) + \lambda_s v_s(t) \\
\dot{x}_s(t) &= z_s(t) + V_s y_s(t) 
\end{align*}
\]

(59)

where \( v_s(t) \) is an external compensation signal and \( \lambda_s \in \mathbb{R}^{n \times p} \) is the feedforward injection map. And \((N_s, \lambda_s)\) is completely controllable.

Subtracting (24) from (59), the error dynamics is

\[
\dot{e}_s(t) = N_s e_s(t) + L_s \eta_s f_s(t) - \lambda_s v_s(t) 
\]

(60)

and the output error dynamics is obtained as

\[
\dot{e}_y(t) = CN_s e_s(t) + CL_s \eta_s f_s(t) - C\lambda_s v_s(t) 
\]

(61)

We desire to find \( \lambda_s \) and \( v_s(t) \) such that the stability of the system is preserved.

According to the similar procedures of subsection 3.1, we have

\[
C\lambda_s v_s(t) = CL_s \eta_s f_s(t) 
\]

(62)

where \( \lambda_s = P_s^{-1} C^T \) and \( P_s \) is a solution of Lyapunov equation, \( V(e^T_s(t)) = e^T_s(t) P_s e_s(t) \).

To recover the equivalent feedforward signal, the continuous approximation signal is written as

\[
v_b^s(t) = W_s e_s^r(t) 
\]

(63)

where \( \delta_s \) is a small positive scalar. From (62) and (63), it follows that

\[
\hat{f}_s(t) = (CL_s \eta_s)^T (CL_s) v_b^s(t) 
\]

(64)

Thus, the reconstruction of sensor faults is achieved from (64).

**Remark 4.** For \( f_s(t) \) in (24), the sensor faults can be reconstructed if

(i) \( CL_s \neq 0 \)

(ii) \( \text{rank}(CL_s \eta_s) = \text{rank} \eta_s \)

Through sliding mode approach, the error dynamics and the output error dynamics are converged to zero, as well as the sensor faults \( f_s(t) \) are reconstructed. Unfortunately, the state errors are perfectly converged to zero is very difficult, since the generalized matrix inverse and the structural problem of error dynamics itself. To obtain improved reconstructing signals, the errors must be minimized or eliminated.

Let’s consider the enhanced form of sliding mode observer with \( \hat{f}_s(t) \) of (64) as

\[
\begin{align*}
\dot{z}_s^*(t) &= N_s z_s^*(t) + L_s y_s(t) + J_s u(t) + L_s \eta_s \hat{f}_s(t) - \lambda_s v_s^*(t) \\
\dot{x}_s^*(t) &= z_s^*(t) + V_s y_s(t) 
\end{align*}
\]

(65)

The error dynamics is determined as same as above procedures

\[
\dot{e}_s^*(t) = N_s e_s^*(t) + L_s \eta_s (f_s(t) - \hat{f}_s(t)) - \lambda_s v_s^*(t) 
\]

(66)

and the output error dynamics is rewritten as

\[
\dot{e}_y^*(t) = CN_s e_s^*(t) + CL_s \eta_s (f_s(t) - \hat{f}_s(t)) - C\lambda_s v_s^*(t) 
\]

(67)

Also, the enhanced continuous feedforward compensation signal is as

\[
v_b^s(t) = W_s e_s^r(t) 
\]

(68)

and the reconstruction of \( f_s^*(t) \) is

\[
\hat{f}_s^*(t) = (CL_s \eta_s)^T (CL_s) v_b^s(t) 
\]

(69)

where \( e_s^r(t) \) is output error signals between (24) and (65),
and $W^s_\ast$ and $\delta^s_\ast$ are a constant diagonal matrix and small constant, respectively. As a result, the reconstruction of sensor faults is achieved as

$$f_s(t) \approx \hat{f}_s(t) + \hat{f}^s_\ast(t)$$

(70)

where the gap between real and reconstructed faults can be minimized. Thus, the reconstruction procedure can show as Fig. 2.

![Fig. 2. The reconstruction procedure of sensor faults.](image)

In the subsection 2.2, the isolation of sensor faults was not achieved because of the derivative signals. The problem can be solved by the reconstruction of the whole faults through the above processes.

**IV. SIMULATION RESULTS**

As an example of a physical system where a descriptor system formulation is applicable, consider the chemical mixing system which has two inputs and three outputs as Fig. 3. If the height, $h_1$ and $h_2$, of the liquid in the mixing tank are constant, then the dynamics model can be found by performing a component mass balance as

$$V_1 \dot{c}_s(t) = c_1 q_1(t) + c_2 q_3(t) - c_3(t)(q_1(t) + q_2(t))$$

$$V_2 \dot{c}_s(t) = c_1 q_1(t) + c_2 q_3(t) - c_3(t)(q_1(t) + q_2(t))$$

where $q_i(t), i = 1, 2, 4$, denotes the flow rates of the input streams, and $c_j, j = 1, 2, 5$, are the concentrations of some chemical in the input streams and output stream respectively. Also, $V_1$ and $V_2$ are the volume of liquid in the mixing tank. For the dynamic model to be valid, the height of liquid in the tank must remain constant. This can be achieved by using a standpipe. If the height is constant, then by an overall mass balance the flow rate for the output stream, $q_3$ and $q_5$, must be equal the total flow rate for the input streams [21].

$$q_3(t) = q_1(t) + q_2(t)$$

$$q_5(t) = q_1(t) + q_4(t)$$

Consider the following definition of input, descriptor and output variables.

![Fig. 3. A chemical mixing tank system.](image)

$$u(t) = [q_1(t), q_4(t)]^T$$

$$z(t) = [c_1(t), q_1(t), c_3(t), q_3(t)]^T$$

$$y(t) = [q_1(t), c_3(t), q_3(t)]^T$$

The reformulation of the dynamic and static equations yields the following nonlinear descriptor model.

$$V_1 \dot{z}_1(t) = c_1 u_1(t) + c_2 q_3(t) - z_1(t) z_2(t)$$

$$V_2 \dot{z}_3(t) = z_1(t) z_2(t) + c_2 u_2(t) - z_3(t) z_4(t)$$

$$z_1(t) = u_1(t) + q_1(t)$$

$$z_2(t) = z_1(t) + u_2(t)$$

where $u_k(t)$ is $k$-th row element of $u(t)$ and $z_m(t)$ is $m$-th row element of $z(t)$.

Suppose the input flow rate is constant with $u_1(t) = \bar{u}_1$ and $u_2(t) = \bar{u}_2$. Then, it follows that the equilibrium values for the descriptor vector and the output are

$$\dot{\bar{z}}_1 = \frac{c_1 \bar{u}_1 + c_2 q_2}{\bar{u}_1 + q_2}$$

$$\dot{\bar{z}}_2 = \bar{u}_1 + q_2$$
\[ \dot{z}_3 = \frac{\dot{z}_1 \dot{z}_3 + c_4 \dot{u}_4}{\dot{z}_2 + \dot{u}_2} \]

\[ \dot{z}_4 = \dot{z}_2 + \dot{u}_2 \]

\[ \hat{y} = [\dot{z}_2, \dot{z}_3, \dot{z}_4]^T \]

Note that \( \dot{z}_1 > 0, \dot{z}_2 > 0, \dot{z}_3 > 0 \) and \( \dot{z}_4 > 0 \). Next, let \( \delta u(t) = u(t) - \dot{u} \), \( \delta z(t) = z(t) - \dot{z} \) and \( \delta y(t) = y(t) - \hat{y} \) denote the variations of \( u(t), z(t), \) and \( y(t) \) from their operating point values respectively. From the linearization at the point, the following linearized descriptor system is yielded.

\[
\begin{bmatrix}
V \delta z(t) = A \delta z(t) + B \delta u(t) \\
\delta y(t) = C \delta z(t)
\end{bmatrix}
\]

where

\[
V = \begin{bmatrix}
V_1 & 0 & 0 & 0 \\
0 & 0 & V_2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
C = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
-\dot{z}_2 & -\dot{z}_1 & 0 & 0 \\
0 & -1 & 0 & 0 \\
\dot{z}_2 & \dot{z}_1 & -\dot{z}_4 & -\dot{z}_3 \\
0 & 1 & 0 & -1
\end{bmatrix},
B = \begin{bmatrix}
c_1 & 0 \\
0 & c_2 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( c_1, c_2, \) and \( c_4 \) are arbitrarily set 4mol/ℓ, 2mol/ℓ, and 1mol/ℓ, and the volume \( V_1 \) and \( V_2 \) is 40ℓ and 50ℓ respectively. Also, \( \dot{u}_t(t) \) and \( \dot{u}_e(t) \) denote 5ℓ/s and 6ℓ/s.

### 4.1 In existing an actuator fault

State matrices of the chemical mixing tank system with an actuator fault are given as

\[
E = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
C = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
A = \begin{bmatrix}
0.3750 & -0.0667 & 0 & 0 \\
0 & -1.0000 & 0 & 0 \\
0 & 0.0533 & -0.5000 & -0.0400 \\
0 & 1.0000 & 0 & -1.0000
\end{bmatrix},
B = \eta_a = \begin{bmatrix}
0.1000 & 0 \[1.0000 & 0 \\
0 & 0.0200 \\
0 & 1.0000
\end{bmatrix}
\]

where we assume that the fault exists in the first valve.

According to the procedure of Section 2, matrices \( E_a^\delta \) and \( C_a^\delta \) are defined by (4)

\[
E_a^\delta = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad C_a^\delta = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The residual signal for first actuator fault is obtained as Fig. 4, from which we can see the fault, \( f_a^\delta(t) \) is generating.

The matrices \( R_a^\delta, V_a^\delta, N_a^\delta, \) and \( K_a^\delta \) of unknown input observer are obtained as

\[
R_a^\delta = \begin{bmatrix}
1.0000 & 0 & 0 & 0 \\
0 & 1.0000 & 0 & 0 \\
0 & 0 & 0.9996 & -0.0200 \\
0 & 0 & -0.0200 & 0.0004
\end{bmatrix}
\]

\[
V_a^\delta = \begin{bmatrix}
1.0000 & 0 & 0 \\
0 & 0.0004 & 0 \\
0 & 0.0200 & 1.0000
\end{bmatrix}
\]

\[
N_a^\delta = \begin{bmatrix}
-0.3750 & 0 & -17.8984 & 3.9399 \\
0 & -2.5000 & 0 & 0 \\
0.2999 & 0 & -5.0124 & 0.5170 \\
-0.0060 & 0 & 0.0472 & -2.7126
\end{bmatrix}
\]

![Fig. 4. The residual signals for first actuator fault.](image-url)
where the matrix $K_1^a$ is gain matrix which eigenvalues of $N_1^a$ are placed at $[-2.5, -2.6, -2.7, -2.8]$. The feedforward injection map $\lambda_s^a$ is given as

$$
\lambda_s^a = \begin{bmatrix}
0.0000 & 0.1237 & -0.1574 \\
5.0000 & -0.0000 & -0.0000 \\
-0.0000 & 10.3489 & -1.0781 \\
-0.0000 & -1.0781 & 5.6324 \\
\end{bmatrix}
$$

By the above matrices and the feedforward signals, the reconstruction results of actuator fault are shown as Fig. 5, from which we can see that the reconstructed result converges to the real fault.

**4.2 In existing a sensor fault**

For above mixing system, we assume that the fault exists in the second sensor, $\eta_s = [0 \ 1 \ 0]^T$.

From (26), the matrices $E_s^a$ and $C_s^a$ are defined as

$$
E_s^a = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad C_s^a = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0.5 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$

The residual signal for $f_s(t)$ is obtained as Fig. 7, from which we can see $f_s(t)$ is existing. The reconstruction result of sensor faults shows as Fig. 8.

Here, matrices $R_s$, $V_s$, $N_s$, and $K_s$ of the unknown input observer are calculated as

$$
R_s = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad V_s = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$

$$
N_s = \begin{bmatrix}
-0.3750 & -0.0140 & -1.4420 & 0.0079 \\
0 & -4.5662 & -0.0179 & -0.3870 \\
0.3000 & 0.0355 & -4.5954 & -0.0274 \\
0 & 0.6130 & 0.0126 & -4.6502 \\
\end{bmatrix}
$$

$$
K_s = \begin{bmatrix}
0.0527 & -1.4420 & 0.0079 \\
-3.5662 & -0.0179 & -0.3870 \\
-0.0179 & -4.0954 & 0.0126 \\
-0.3870 & 0.0126 & -3.6502 \\
\end{bmatrix}
$$

![Fig. 5. The reconstruction result of first actuator fault.](image)

![Fig. 6. The results of output error dynamics.](image)

![Fig. 7. The residual signals for sensor fault.](image)
From these simulation results, the feedforward injection maps and compensation signals are properly obtained to guarantee the stability of the state error dynamics and the existence of sliding mode. Also, we can see that the reconstruction as well as the detection and isolation of each fault are appropriately achieved through the unknown input observers and the feedforward signals.

V. CONCLUSION

This paper proposed FDI approaches for descriptor systems with actuator faults and sensor faults, respectively. Also, this work attempted the reconstruction of each fault. In existing an actuator faults, the FDI problem was solved by the unknown input observer decoupling the influence of the remaining actuator faults. Moreover, the fault reconstruction was achieved from sliding mode observer including feedforward injection map and compensation signal. Whereas, in existing sensor faults, the fault detection was only achieved from the unknown input observer, since the derivative signals of sensor faults generate in the error dynamics. Therefore, the isolation of sensor faults were achieved by reconstructing all of sensor faults. Furthermore, in order to obtain improved reconstruction results, the design of enhanced sliding mode observer added previously reconstructed result was proposed. From simulation results for the chemical mixing tank with standpipe, the effectiveness of proposed approaches were shown. Through this work, if the existence conditions of sliding mode observer were satisfied, the reconstruction of faults can be easily achieved for descriptor systems as well as general linear systems.

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