ROBUST FAULT DETECTION AND ISOLATION FOR UNCERTAIN LINEAR RETARDED SYSTEMS

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ABSTRACT

A robust fault detection and isolation scheme is proposed for uncertain continuous linear systems with discrete state delays for both additive and multiplicative faults. Model uncertainties, disturbances and noises are represented as unstructured unknown inputs. The proposed scheme consists of a Luenberger observer for fault detection and a group of adaptive observers, one for each class of faults, for fault isolation. The threshold determination and fault isolation are based on a multi-observer strategy. Robustness to model uncertainties and disturbances can be guaranteed for the scheme by selecting proper thresholds. All the signals, i.e., the fault estimate and the state and output estimation errors of each isolation observer can be shown to be uniformly bounded, and the estimate of the fault by the matched observer is shown to be satisfactory in the sense of extended $L_2$ norm. Furthermore, the sensitivity to fault and the fault isolability condition are analyzed also in the paper. Simulations of a heating process for detecting and isolating an actuator gain fault and an additive fault show the proposed scheme is effective.

Keywords: Fault detection and isolation, fault identification, linear retarded time-delay systems, uncertain systems, adaptive observers.

I. INTRODUCTION

Many fault diagnosis methods have been developed for dynamic systems without delays. Like control system design, fault diagnosis faces the challenge of time delay as it widely exists in process industries.

For fault diagnosis of linear retarded systems, Yang and Saif [1] proposed a diagnosis scheme for actuator and sensor faults using an unknown input observer and a technique of input estimation. If the influence of disturbances cannot be completely decoupled, Liu and Frank [2] transformed the design of fault detection filter into a two-step optimization problem, i.e., first parameterized the filter gain matrix guaranteeing a certain $H_\infty$ disturbance-attenuation performance of the diagnostic residual, then selected the best one maximizing the sensitivity of the residual to faults. Following the same way, Jiang et al. [3] extended Liu and Frank’s work [2] to discrete time-delay systems. Ding et al. [4] reformulated the previous two-step optimization problem into a $H_\infty$ model-matching problem, where the reference model maximizing the sensitivity of the residual to faults. Then, Zhong et al. [5] combined $H_\infty$ filtering with eigenvalue structure assignment to further enhance the diagnostic robustness to disturbances. In these works, only structured additive disturbances and faults like $Bu(t)$ were considered, while many faults represented by undesired parametric changes couldn’t be transformed into this form. The additive structured form may also be inap-
Among fault diagnosis methods for non-delay processes, adaptive techniques have been widely adopted, where a fault can not only be detected but also be estimated, and the estimation result can be further used for fault accommodation, see [8-14] and the references therein. An on-line learning scheme has been developed to detect and estimate faults by Polycarpou and Helmicki [10] for a class of nonlinear systems with unstructured faults and disturbances. This was extended by Vemuri and Polycarpou [11] to certain input-output systems using a particular adaptive observer with an auxiliary filtering transformation [15]. The convergence of the above fault estimates can not be guaranteed as that the persistency of excitation condition couldn’t be ensured. Zhang and Polycarpou [12] proposed further a fault isolation scheme with multi-observer strategy [16]. Similarly, Xu and Zhang [14] proposed a residual generator design method using adaptive observers, under the assumption of some persistently exciting signals, in a stochastic framework.

In this paper, we extend the fault detection and isolation schemes of [12] and [14] by use of a robust design and making the method applicable to dynamic processes with multiple discrete state delays. Abrupt faults are taken as unknown inputs premultiplied by time-varying gain matrices, which can represent a class of parametric or additive faults. Model uncertainties, disturbances and noises are all taken as unstructured bounded unknown inputs. The proposed robust fault detection and isolation scheme consists of a detection observer and a group of isolation observers in a newly developed adaptive observer form [7], which are activated upon the detection of a fault. Each observer has an adaptive threshold in the decision-making. Fault isolation is conducted through the analysis of the residual of each isolation observer. The fault estimated by the matched observer, using the correct fault direction information, may be further explored in fault accommodation schemes such as fault-tolerant control. With appropriate thresholds, the proposed fault detection and isolation scheme can be shown robust to model uncertainties and disturbances. It can also be shown that the fault estimate and the state and output estimation errors of each isolation observer are all uniformly bounded. Furthermore, the sensitivity to faults and the fault isolability condition is analyzed.

The organization of this paper is as follows. Following the introduction, preliminary knowledge is presented in Section II. The problem is formulated in Section III. The main results of the paper are presented in Section IV, where the design of fault detection observer and its threshold, the analysis of sensitivity to fault are presented as the first part; the design of fault isolation observers and their adaptive thresholds, the analysis of the estimation performance and the isolability condition are presented as the second part. Simulation applications of the proposed fault detection and isolation scheme to a heating process are given in Section V. Finally, Section VI concludes the paper.

Notations. Denotes by $L_{2}(\mathbb{R}^n, \mathbb{R}^n)$ the space of Lebesgue square integrable functions defined on $[a, b]$ with values in $\mathbb{R}^n$. Then, $H = \mathbb{R}^n \times L_\infty([-d, 0], \mathbb{R}^n)$ [17] for $d \geq 0$ stands for the Hilbert space with inner product $\langle u, u \rangle = v^T v + \int_{-d}^{0} \phi^T (\theta) \phi (\theta) d\theta$, where $u = (v, \phi)$ $\in H$, and the induced norm is $||u||^2 = v^T v + \int_{-d}^{0} \phi^T (\theta) \phi (\theta) d\theta$. $|| \cdot ||$ denotes the Euclidean vector norm or induced matrix 2-norm. We say that $\sigma (t) \in L_{\infty}$ if $|| \sigma (t) ||_\infty$ exists, where $|| \sigma (t) ||_\infty := \sup_{t \in \mathbb{R}^+} | \sigma (t) |$. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are, respectively, the minimal and maximal eigenvalues of a symmetric matrix. $I_n$ and $0_n$ are, respectively, the $n \times n$ identity matrix and zero matrix. $P > 0$ denotes a symmetric and positive matrix $P$.

## II. PRELIMINARIES

This section presents some necessary background knowledge on the solution of retarded type delay differential equation represented by

$$
\dot{z}(t) = \sum_{i=0}^{\bar{n}} \bar{A}_i z(t - d_i) + \bar{B} v(t)
$$

where $\dot{z}(t) \in \mathbb{R}^n$, $v(t) \in \mathbb{R}^r$ and $0 = d_0 < d_1 < \cdots < d_{\bar{n}} = d < \infty$. $\bar{A}_0, \ldots, \bar{A}_{\bar{n}}, \bar{B}$ are real matrices with proper dimensions. The initial state satisfies $z(t_0 + \theta) = \phi(\theta)$ for $\theta \in [-d, 0]$, where $(\phi(0), \phi(\theta)) \in H$ and $t_0 \geq 0$.

The solution of Eq. (1) is [18]

$$
z(t) = \Psi(t - t_0) z(t_0) + \sum_{i=1}^{\bar{n}} \int_{t_i - d_i}^{t} \Psi(t - \tau) \bar{A}_i z(\tau - d_i) d\tau + \int_{t_0}^{t} \Psi(t - \tau) \bar{B} v(\tau) d\tau, \quad t \geq t_0
$$

where $\Psi(t) \in \mathbb{R}^{n \times r}$, with $\Psi(0) = I_n$ and $\Psi(\theta) = 0_n$ for all $\theta \in [-d, 0)$, is the fundamental matrix solution of the homogeneous equation

$$
\dot{z}_h(t) = \sum_{i=0}^{\bar{n}} \bar{A}_i z_h(t - d_i).
$$

For Eq. (3) with $z_h(t_0 + \theta) = \phi(\theta)$ where $\theta \in [-d, 0]$, an exponential estimate [19] of its solution can be derived directly following the result of Mondié and Kharitonov [20].
Lemma 1. If there exist symmetric matrices $P > 0$, $Q_1, \ldots, Q_q \in \mathbb{R}^{n \times n}$ and a scalar $\rho > 0$, such that the following inequality holds
\[
\text{diag}\left\{\sum_{i=1}^{q} Q_i + 2\rho P, -e^{-2\rho\tau} Q_1, \ldots, -e^{-2\rho\tau} Q_q\right\} + \mathcal{N}^T P \mathcal{E} + \mathcal{E}^T P \mathcal{N} < 0,
\]
with $\mathcal{N} = [\overline{A}_0, \overline{A}_1, \ldots, \overline{A}_q]$ and $\mathcal{E} = [I_n, 0, \ldots, 0_n]$, then the solution of Eq. (3) satisfies the inequality
\[
|z_h(t)| \leq e^{\rho t - \rho t_0} \|\phi(0)\|_n + \int_{0}^{t} e^{\rho \tau - \rho t_0} \|\mathcal{B}(\tau)\|_n d\tau,
\]
$t \geq t_0$
where $\rho = \frac{\alpha_1}{\alpha_2}, \alpha_2 = \max\{\lambda_{\max}(P), \sum_{i=1}^{q} \lambda_{\max}(Q_i)\}$, and $\alpha_1 = \lambda_{\min}(P)$.

From its proof, we can find that Eq. (3) is asymptotically stable if this lemma holds for $\rho = 0$.

Lemma 2. If there exists an exponential estimate of the solution of retarded type delay differential Eq. (1) can be represented as
\[
|z(t)| \leq \mu e^{\rho(t - t_0)} \|\phi(0)\|_n + \int_{0}^{t} \mu e^{\rho(\tau - t_0)} \|\mathcal{B}(\tau)\|_n d\tau,
\]
t $\geq t_0$
where $\mu$ and $\rho$ are derived in Lemma 1.

The proof of this lemma is presented in the appendix.

III. PROBLEM FORMULATION

Consider the following system
\[
\begin{align*}
\dot{x}(t) &= \sum_{j=0}^{q} A_j x(t - d_j) + Bu(t) \\
&+ Z(y(t), \ldots, y(t - d_q), u(t)) B(t) f \\
&+ \eta_h(x(t), \ldots, x(t - d_q), u(t), t) \\
y(t) &= C_0 x(t) + Du(t) + \eta_s(x(t), u(t), t)
\end{align*}
\]
where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$ and $y(t) \in \mathbb{R}^m$ are, respectively, the state, input and output of the system. $A_0, \ldots, A_q, B, C_0, D$ are known constant real matrices with proper dimensions. $0 = d_0 < d_1 < \cdots < d_q = d < \infty$ are constant delays. The initial condition is $x(0) = \phi(0)$ for $\theta \in [-d, 0]$, where $\phi(0), \Phi(0) \in \mathcal{H}$.

$Z(y(t), \ldots, y(t - d_q), u(t)) B(t) f$ denotes the possible fault, where $f \in \mathbb{R}^h$ is an unknown constant vector. $B(t)$ is a time profile function defined by
\[
B(t) := \text{diag}\{\beta_1(t - T_1), \ldots, \beta_h(t - T_h)\}
\]
with $\beta_j(t - T_j) = \begin{cases} 0 & \text{if } t < T_j \\ 1 & \text{otherwise} \end{cases}$ for $j = 1, \ldots, h$.

If elements of $f$ are all zero except for the $j$th element, then there is a single fault starting to influence the system at the unknown time $T_j$. In this paper, we only consider abrupt faults. $Z(y(t), \ldots, y(t - d_q), u(t)) \in \mathbb{R}^{n \times n}$ is a known matrix function, which is written in a compact form, $Z(Y_d(t), u(t))$, in the remaining parts of the paper for convenience, where $Y_d(t) := \{y(t), \ldots, y(t - d_q)\}$. The $j$th column of $Z(Y_d(t), u(t))$ is represented by $Z_j(Y_d(t), u(t))$.

Remark 1. This fault description is similar to those of [14] and [12], which can represent a class of multiplicative faults related directly to the measurable output and input. Specially, when $Z(Y_d(t), u(t))$ doesn’t depend on $Y_d(t)$ and $u(t)$, $Z(Y_d(t), u(t)) B(t) f$ represents additive faults.

$\eta_h(x(t), \ldots, x(t - d_q), u(t), t)$ and $\eta_s(x(t), u(t), t)$ are the unstructured unknown inputs representing model uncertainties, noises, and unknown disturbances. $\eta_h(x(t), \ldots, x(t - d_q), u(t), t)$ is written as $\eta_s(X_d(t), u(t), t)$ for convenience, where $X_d(t) := \{x(t), \ldots, x(t - d_q)\}$.

For system (5), the following assumptions must be made:

Assumption 1. $Z(Y_d(t), u(t)) \in \mathcal{L}_\infty$ for all $(x(t), u(t), y(t)) \in \mathcal{X} \times \mathcal{U} \times \mathcal{Y}$, where $\mathcal{X} \subset \mathbb{R}^n$ is a compact domain of interest, and $\mathcal{U} \subset \mathbb{R}^l$ and $\mathcal{Y} \subset \mathbb{R}^m$ are the compact sets representing the admissible inputs and outputs, respectively.

Assumption 2. For all $(x(t), u(t), y(t)) \in \mathcal{X} \times \mathcal{U} \times \mathcal{Y}$ and $t \geq 0$ there exist
\[
|\eta_h(X_d(t), u(t), t)| \leq \overline{\mathcal{H}}_i(Y_d(t), u(t), t) < \infty
\]
and
\[
|\eta_s(x(t), u(t), t)| \leq \overline{\mathcal{H}}_s(Y_d(t), u(t), t) < \infty.
\]

Assumption 3. $x(t) \in \mathcal{L}_\infty$ for all $t \geq 0$ even under the influence of the faults.

Assumption 4. The faults considered in this paper belong to $h$ known classes, where the $j$th class of fault ($j \in \{1, \ldots, h\}$) is represented by $Z_j(Y_d(t), u(t)) \beta_j(t - T_j) f_j$ with direction vector $Z_j(Y_d(t), u(t)) \beta_j(t - T_j)$ available and magnitude in a finite interval, i.e., there exist $0 < r_j < \infty$ and $c_j$ such that $|f_j - c_j| \leq r_j$.

Assumption 5. Only the case that a single fault occurs at a time is considered.
Remark 2. Assumption 1 is made for ensuring the boundedness of estimation signals. Assumption 2 is used to derive thresholds for distinguishing the faults from the disturbances. When function $\overline{r}_i(y(t), u(t), t)$ and $\overline{r}_j(y(t), u(t), t)$ are unavailable, the special forms of constant upper bounds can be used instead, which may introduce some conservativeness. This paper assumes that a finite disturbance set is known from the prior process knowledge and all faults are within the set. The magnitudes of each class of faults are unknown except for the possible bounds. Assumption 5 doesn’t lose much generality, since the probability of multiple faults occurring at a specific time is much less than that of a single fault.

Now the problem of this paper is to detect, isolate, and estimate the size of the fault for system described by (5).

IV. ROBUST FAULT DETECTION AND ISOLATION SCHEME

To solve the problem, the following scheme is proposed to incorporate a novel adaptive observer design [7] to the well known generalized observer strategy [16].

1. A fault detection observer (FDO) is utilized to monitor the system and to detect whether a fault has occurred.
2. Upon the detection of a fault, a group of fault isolation observers (FIOs) are activated. Each observer generates its estimate of the fault. The isolation of the fault is made from the residual analysis for each observer.
3. The matched isolation observer then derives the correct size estimate of the fault for on-line fault approximation.

The proposed fault detection and isolation scheme is illustrated in Fig. 1.

The fault detection observer used is a Luenberger observer designed for the nominal time delay system without disturbances and faults. A fault is assumed to have occurred when the residual of FDO surpasses its threshold at time $T_D > 0$. The fault isolation observers consist of $h$ adaptive observers each of which corresponds to a class of fault. Only one of them, the one using the correct fault direction information with the minimal residual, will be fired as the matched observer for fault estimation. At time $T_I > T_D$, all residuals of the $k$th observer for $k \in \{1, \ldots, h\}$ are greater than their corresponding thresholds while the residual of the $j$th observer is always smaller than its threshold, this corresponds to the occurring of $j$th class fault. We name $T_D$ and $T_I$, respectively, as fault detection time and fault isolation time.

So the main tasks of the proposed scheme are the design of the fault detection observer, the fault isolation observers and their corresponding adaptive thresholds.

Remark 3. If the fault estimates are accurate, faults can be isolated directly based on the estimation results. However, with model uncertainties and disturbances, fault estimates can’t be guaranteed to converge to their real values especially when multiple parameters are estimated simultaneously [7]. Persistent excitation (PE) can result in a better estimation performance, but this is a rather strict condition for application. Multi-observer strategy is proposed in this paper for fault isolation so as to ensure the matched observer with a good estimate.

4.1 Robust fault detection observer

The fault detection observer is designed as

$$
\dot{\hat{x}}_k(t) = \sum_{i=0}^{q} A_i \hat{x}_i(t-d_i) + Bu(t) + K_0(y(t) - \hat{y}_0(t))
$$

$$
\dot{\hat{y}}_0(t) = C_0 \hat{x}_0(t) + Du(t)
$$

(6)

where $\hat{x}_k(t) \in \mathbb{R}^n$ and $\hat{y}_0(t) \in \mathbb{R}^m$ are the estimates of state and output, respectively. $\hat{x}_k(0) = 0$ for $0 \in [-d_0, 0]$. $K_0$ is the gain matrix that should guarantee the following system

$$
\dot{e}_{00}(t) = (A_0 - K_0 C_0) e_{00}(t) + \sum_{i=1}^{q} A_i e_{i0}(t - d_i)
$$

(7)

asymptotically stable, where $e_{00}(t) = x(t) - \hat{x}_0(t)$. This design can be realized by the following lemma that ensures the system stable delay-independently [21].

Lemma 3. If there exist symmetric matrices $P > 0$, $Q_1, \ldots, Q_q \in \mathbb{R}^{m \times m}$ and a matrix $R \in \mathbb{R}^{m \times n}$, such that the following linear matrix inequality

$$
\text{diag}\left\{-RC_0 - C_0^T R^T + \sum_{i=1}^{q} Q_i, -Q_1, \ldots, -Q_q\right\} + N^T PE + E^T P N < 0
$$

with $N = [A_0, A_1, \ldots, A_q]$ and $E = [I_n, 0_n, \ldots, 0_n]$ holds,
then system (7) will be asymptotically stable delay-independently and \( K_0 = P^{-1} R \).

In the following, it is supposed that Lemma 3 holds with a \( K_0 \) ensuring the stability of system (7).

System (7) can be considered as the error dynamics between system (5) and fault detection observer (6) when the unknown disturbances and faults are zero. However, in reality, the influences of unknown disturbances will cause the residual of observer (6) deviates from zero. To make the fault detection robust to the unknown disturbances, a proper threshold must be used to prevent false detection.

From system (5) and fault detection observer (6), the following error dynamics can be derived.

\[
\begin{align*}
\dot{e}_0(t) &= (A_0 - K_0 C_0) e_0(t) + \sum_{i=1}^{n} A_i e_0(t - d_i) \\
+ \eta_i (X_i(t), u(t), t) - K_0 \eta_i (x(t), u(t), t) \\
\end{align*}
\]

\[
\begin{align*}
e_0(t) &= C_0 e_0(t) + \eta_i (x(t), u(t), t)
\end{align*}
\]

where \( e_0(t) = x(t) - \hat{x}_0(t) \) and \( \hat{e}_0(t) = y(t) - \hat{y}_0(t) \) are, respectively, the state and output estimation error. \( \hat{e}_0(t) \) is the derived residual for fault detection. \( \hat{e}_0(t) = x(\theta) \) for \( \theta \in [-d, 0] \) since \( \hat{e}_0(0) = 0 \). Based on formula (2), its solution for \( t \geq 0 \) can be derived as

\[
\begin{align*}
\hat{e}_0(t) &= e_0(0) + \sum_{i=1}^{n} \int_{0}^{t} \dot{e}_0(\tau) A_i \dot{e}_0(t - d_i) d\tau \\
+ \eta_i (x(t), u(t), t)
\end{align*}
\]

\[
\begin{align*}
e_0(t) &= C_0 e_0(t) + \eta_i (x(t), u(t), t)
\end{align*}
\]

\[
\begin{align*}
e_0(t) &= C_0 \left[ \Phi(t) x(0) + \sum_{i=1}^{n} \int_{0}^{t} \Phi(\tau) A_i x(\tau - d_i) d\tau \\
+ \eta_i (x(t), u(t), t) \right] \\
+ \eta_i (x(t), u(t), t)
\end{align*}
\]

\[
\begin{align*}
e_0(t) &= C_0 \left[ \Phi(t) x(0) + \sum_{i=1}^{n} \int_{0}^{t} \Phi(\tau) A_i x(\tau - d_i) d\tau \\
+ \eta_i (x(t), u(t), t) \right] \\
+ \eta_i (x(t), u(t), t)
\end{align*}
\]

\[
\begin{align*}
\int_{\tau}^{\infty} C_0 \Phi(t - \tau) \sum_{i=1}^{n} \int_{0}^{t} \Phi(\tau) A_i x(\tau - d_i) d\tau
\end{align*}
\]

\[
\begin{align*}
\int_{t_j}^{\infty} C_0 \Phi(t - \tau) Z_j (Y_j(t), u(t)) f_j dt
\end{align*}
\]

Theorem 1. The fault of class \( j \) is detectable if there exists a finite time \( t_j > 0 \) such that the following condition holds.

\[
\begin{align*}
\int_{t_j}^{\infty} C_0 \Phi(t - \tau) Z_j (Y_j(t), u(t)) f_j dt 
\end{align*}
\]

Proof. In the presence of \( j \)th class fault, the error dynamics becomes

\[
\begin{align*}
\dot{e}_0(t) &= (A_0 - K_0 C_0) e_0(t) + \sum_{i=1}^{n} A_i e_0(t - d_i) \\
+ \eta_i (X_i(t), u(t), t) - K_0 \eta_i (x(t), u(t), t) \\
+ Z_j (Y_j(t), u(t)) \beta_j (t - T_j) f_j \\
e_0(t) &= C_0 e_0(t) + \eta_i (x(t), u(t), t)
\end{align*}
\]

then based on formula (2) the output estimation error for \( t \geq 0 \) obeys

\[
\begin{align*}
e_0(t) &= C_0 \left[ \Phi(t) x(0) + \sum_{i=1}^{n} \int_{0}^{t} \Phi(\tau) A_i x(\tau - d_i) d\tau \\
+ \eta_i (x(t), u(t), t) \right] \\
+ \eta_i (x(t), u(t), t)
\end{align*}
\]

\[
\begin{align*}
\int_{t_j}^{\infty} C_0 \Phi(t - \tau) Z_j (Y_j(t), u(t)) f_j dt
\end{align*}
\]

It is obvious that

\[
\begin{align*}
\left| e_0(t) \right| &\geq \left| \int_{t_j}^{\infty} C_0 \Phi(t - \tau) Z_j (Y_j(t), u(t)) f_j dt \right| \\
- \left| \eta_i (x(t), u(t), t) \right| \\
+ \left| \int_{t_j}^{\infty} C_0 \Phi(t - \tau) \eta_i (X_i(t), u(t), t) - K_0 \eta_i (x(t), u(t), t) dt \right| \\
+ \left| \int_{t_j}^{\infty} C_0 \Phi(t - \tau) \eta_i (Y_i(t), u(t), t) dt \right| \\
+ \left| \int_{t_j}^{\infty} C_0 \Phi(t - \tau) \eta_i (x(t), u(t), t) dt \right|
\end{align*}
\]
Considering the determination of $\mathbb{T}_h(t)$, $|e_h(T_j + t_j)| \geq \mathbb{T}_h(t + t_j)$ if condition (10) holds. Then the fault is detectable.

**Remark 4.** This theorem gives the worst case for a fault to be detectable, that is when the fault influence on the residual is no less than $2\mathbb{T}_h(t)$. Condition (10) is only sufficient, not necessary, i.e., a fault may still be detectable even if it isn’t satisfied. This theorem presents also a measure of the fault detection time; $T_f + t_j$ can be considered as an upper bound of the detection time for the $j$th class faults.

### 4.2 Robust fault isolation observers

After a fault is detected, a group of $h$ fault isolation observers are enabled to work for isolating and approximating the fault. For the $j$th class of faults, $j \in \{1, \ldots, h\}$, adaptive observer $j$ is designed as

$$
\dot{x}_j(t) = \sum_{i=1}^{q} A \dot{x}_j(t - d) + B u(t) + Z_j(Y_j(t), u(t)) \dot{f}_j(t) + K_j(y(t) - \dot{y}_j(t)) - \Omega_j(t) \dot{f}_j(t)
$$

$$
\dot{y}_j(t) = C \dot{x}_j(t) + D u(t)
$$

with

where $\dot{x}_j(t) \in \mathbb{R}^n$ and $\dot{y}_j(t) \in \mathbb{R}^m$ are, respectively, the estimates of state and output. The initial condition is $\dot{x}_j(T_0 + \theta) = 0$ for $\theta \in [-d, 0]$, where $T_0$ is the time instance when the fault is detected. $K_j \in \mathbb{R}^{m \times n}$ is the gain matrix, which is designed as $K_j = K_0$ in this paper. $\Omega_j(t) \in \mathbb{R}^n$ is the state of the following auxiliary filter

$$
\dot{\Omega}_j(t) = (A_0 - K_j C_0) \Omega_j(t)
$$

with the initial condition $\dot{\Omega}_j(T_0 + \theta) = 0$ for $\theta \in [-d, 0]$.

$\dot{f}_j(t) \in \mathbb{R}^n$ is the estimate of the $j$th class fault. Using Lyapunov synthesis, the following adaptive law is designed for $\dot{f}_j(t)$

$$
\dot{\gamma}_j(t) = \mathcal{P}[-\gamma_j(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t))]
$$

with

$$
\mathcal{P}[-\gamma_j(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t))]
$$

$$
= -\gamma_j(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t)) (1 - \mathcal{I}_j(t))
$$

which is utilized to prevent the phenomenon of parametric drifting under the influence of model uncertainties [22]. It is easy to prove that if $|\dot{f}_j - c_j| \leq r_j$ and $|\dot{f}_j(T_0) - c_j| \leq r_j$ then $|\dot{f}_j(T_0 + t) - c_j| \leq r_j$ for all $t \geq 0$.

The following theorem gives estimation performance of the fault isolation observers.

**Theorem 2.** In the presence of a $j$th class fault, the scheme of (11)-(13) constitutes a robust fault estimation system guaranteeing that the state estimation error $e_j(t)$, the output estimation error $e_k(t)$, and the fault estimate $\dot{f}_j(t)$ of the isolation observer $k$, $j$, $k \in \{1, \ldots, h\}$, are uniformly bounded; and for any finite $k > 0$, there exist a constant $\kappa$ and a bounded function $\xi(t)$ depending only on the unknown disturbances such that the output estimation error of the isolation observer $j$ satisfies

$$
\int_{T_0}^{T_0 + \tau} |e_j(t)|^2 dt \leq \kappa + 4 \int_{T_0}^{T_0 + \tau} |\xi(t)|^2 dt
$$

where $T_0$ is the fault detection time.

**Proof.** In the presence of a fault of class $j$, it can be derived from system (5) and the $k$th observer (11) that

$$
\dot{\xi}_k(t) = (A_k - K_k C_k) \xi_k(t) + \sum_{i=1}^{q} A_i (t - d_i) - Z_k(Y_k(t), u(t))
$$

$$
+ \eta_k(X_k(t), u(t), t) - K_k \eta_k(x(t), u(t), t)
$$

$$
+ Z_k(Y_k(t), u(t)) f_j - Z_k(Y_k(t), u(t)) \dot{f}_j(t)
$$

$$
+ \Omega_k(t) \dot{f}_j(t) + \sum_{i=1}^{q} A_i \Omega_k(t - d_i) \dot{f}_j(t - d_i))
$$

$$
\xi_k(t) = C_0 \xi_k(t) + \eta_k(x(t), u(t), t)
$$

where $e_k(t) = x(t) - \hat{x}_k(t)$ and $e_k(t) = y(t) - \hat{y}_k(t)$. The initial condition of the error system satisfies $e_k(T_0 + \theta) = x(T_0 + \theta)$ for $\theta \in [-d, 0]$ since $\hat{x}(T_0 + \theta) = 0$. By introducing an auxiliary variable $\tilde{e}_k(t) := e_k(t) + \Omega_k(t) \dot{f}_j(T_0 - \theta)$ and using the filtering transformation (12) [15], we can obtain

$$
\dot{\tilde{e}}_k(t) = \mathcal{P}[-\tilde{e}_k(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t)) (1 - \mathcal{I}_j(t))]
$$

where $\mathcal{P}[-\tilde{e}_k(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t)) (1 - \mathcal{I}_j(t))]

= -\tilde{e}_k(C_0 \Omega_j(t))^T (y(t) - \dot{y}_j(t)) (1 - \mathcal{I}_j(t))
\[
\begin{aligned}
\dot{\bar{e}}_k(t) &= (A_0 - K_\Omega C_0)\bar{e}_k(t) + \sum_{i=1}^q A_i \bar{e}_i(t - d_i) \\
&\quad + \eta_j(X_d(t), u(t), t) - K_\eta \eta_j(x(t), u(t), t) \\
\dot{e}_j(t) &= \bar{e}_j(t) - \Omega_j(t)f_j(t) + \Omega_j(t)\dot{f}_j(t) \\
\dot{e}_k(t) &= C_0 \bar{e}_k(t) - C_0 \Omega_j(t)f_j \\
&\quad + C_0 \Omega_j(t)\dot{f}_j(t) + \eta_j(x(t), u(t), t) \\
\end{aligned}
\]  

(16)

where \( \bar{e}_k(t) = x(T_\delta + \theta) \) for \( \theta \in [-d, 0] \) since \( \Omega_j(T_\delta + \theta) = \Omega_j(T_\delta + 0) = 0 \). As before, the projection operator (14) guarantees that \( \hat{f}_j(t) \in \mathcal{L}_2 \), and from assumption 4, \( f_j(t) \in \mathcal{L}_2 \). Since \( K_k = K_0 \), the system of \( \bar{e}_k(t) \) in (16) is asymptotically stable with zero input. So under assumption 2, there is \( \bar{e}_k(t) \in \mathcal{L}_2 \). From assumption 1 and (12), \( \Omega_j(t), \Omega_j(t) \in \mathcal{L}_2 \) can also be derived. Thus from (16), conclusions can be drawn that \( e_j(t) \), \( e_k(t) \in \mathcal{L}_2 \), and the first part of this theorem holds.

Now, consider the case of first \( k = j \). The following proof follows the same procedure in [11]. Let \( \bar{e}_j(t) = \zeta_j(t) + z_2(t) \) and decompose the dynamics of \( \bar{e}_j(t) \) into two parts

\[
\begin{aligned}
\dot{\zeta}_j(t) &= (A_0 - K_\Omega C_0)\zeta_j(t) + \sum_{i=1}^q A_i \zeta_i(t - d_i) \\
&\quad + \eta_j(X_d(t), u(t), t) - K_\eta \eta_j(x(t), u(t), t) \\
\zeta_j(T_\delta + \theta) &= 0, \quad \theta \in [-d, 0] \\
\dot{z}_2(t) &= (A_0 - K_\Omega C_0)z_2(t) + \sum_{i=1}^q A_i z_2(t - d_i) \\
\zeta_2(T_\delta + \theta) &= x(T_\delta + \theta), \quad \theta \in [-d, 0] \\
\end{aligned}
\]

Then select the following Lyapunov function candidate

\[
V(t) = \frac{1}{\gamma_j} \hat{f}_j(t)^2 + \frac{4}{3} \int_{\tau}^{T_\delta} |C_0 \zeta_2(\tau)|^2 d\tau
\]

where \( \hat{f}_j(t) = f_j - \hat{f}_j(t) \) and differentiate \( V(t) \) along (13)

\[
\begin{aligned}
\dot{V}(t) &= -2 \gamma_j \mathcal{P}[\gamma_j(C_0 \Omega_j(t))\dot{f}_j(t) - \frac{4}{3} |C_0 \zeta_2(t)|^2] \\
&= 2(C_0 \Omega_j(t))^T \dot{f}_j(t) \hat{f}_j(t) \\
&\quad - 2(C_0 \Omega_j(t))^T \dot{f}_j(t) \overline{f}_j(t) + \frac{4}{3} |C_0 \zeta_2(t)|^2 \\
\end{aligned}
\]

From the definition of the projection operator (14), it can be proved that \( 2(C_0 \Omega_j(t))^T e_j(t) \hat{f}_j(t) \geq 0 \), therefore

\[
\dot{V}(t) \leq 2 e_j(t)^T C_0 \Omega_j(t) \hat{f}_j(t) - \frac{4}{3} |C_0 \zeta_2(t)|^2
\]

From the third equation in (16), there is

\[
\begin{aligned}
\dot{V}(t) &\leq -2 |e_j(t)|^2 + 2e_j^2(t) (C_0 \bar{e}_j(t) + \eta_j(x(t), u(t), t)) \\
&\quad - \frac{4}{3} |C_0 \zeta_2(t)|^2 \\
&= -2 |e_j(t)|^2 + 2e_j^2(t) (C_0 \zeta_j(t) + \eta_j(x(t), u(t), t)) \\
&\quad + 2e_j^2(t) C_0 \zeta_2(t) - \frac{4}{3} |C_0 \zeta_2(t)|^2
\end{aligned}
\]

(17)

After completing squares, we can obtain that

\[
\begin{aligned}
\dot{V}(t) &\leq -\frac{1}{4} |e_j(t)|^2 + |C_0 \zeta_j(t) + \eta_j(x(t), u(t), t)|^2 \\
&\leq \kappa + 4 \int_{T_\delta}^{T_\delta + t_f} |C_0 \zeta_j(t) + \eta_j(x(t), u(t), t)|^2 dt
\end{aligned}
\]

where \( \kappa = \sup_{\tau > 0} 4\left(|V(T_\delta) - V(T_\delta + t_f)|\right) \). From (16) and formula (2), the output estimation error for \( j \) class fault for \( t \geq T_\delta \), can be derived as

\[
\begin{aligned}
e_j(t) &= C_0 \left\{ \Phi(t - T_\delta)x(T_\delta) + \sum_{i=1}^q \int_{T_\delta}^{T_\delta + t_f} \Phi(t - \tau)A_i x(\tau - d_i) d\tau \\
&\quad + \int_{T_\delta}^{T_\delta + t_f} \Phi(t - \tau)[\eta_j(X_d(\tau), u(\tau), \tau) - K_\eta \eta_j(x(\tau), u(\tau), t)] d\tau \\
&\quad - C_0 \Omega_j(t) \hat{f}_j(t) + \eta_j(x(t), u(t), t) \right\}
\end{aligned}
\]

Then from the fact that the error dynamics (16) is asymptotically stable and Lemma 2, an estimate of the residual of observer \( j \), \( |e_j(t)| \), for \( t \geq T_\delta \)
\[ \varepsilon_j(t) \leq \mu |C_0| \|x(T_D + \theta)\| e^{-\rho j(T_D + \theta)} \]
\[ + \int_{T_D}^{T+\theta} \mu |C_0| e^{\rho j(t)} [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau + \int_{T_D}^{T+\theta} \mu |C_0| [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau \]
\[ + \tilde{\eta}_j(y(t), u(t), t) \] can be obtained. Considering that \( \tilde{f}_j(t) \) is bounded and will decay to a small value with the matched observer, a practical threshold \( \overline{\varepsilon}_j(t) \) can be selected for isolation observer \( j \),
\[ \overline{\varepsilon}_j(t) := \left| \mu |C_0| a_j e^{-\rho j(T+\theta)} \right| \int_{T_D}^{T+\theta} \mu |C_0| e^{\rho j(t)} [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau \]
\[ + \tilde{\eta}_j(y(t), u(t), t) \geq \mu |C_0| a_j e^{-\rho j(T+\theta)} \int_{T_D}^{T+\theta} \mu |C_0| e^{\rho j(t)} [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau \]
\[ + \tilde{\eta}_j(y(t), u(t), t) \] for \( t \geq T_D \). It is obvious that
\[ \left| \varepsilon_j(t) \right| \geq \mu |C_0| a_j e^{-\rho j(T+\theta)} \int_{T_D}^{T+\theta} \mu |C_0| e^{\rho j(t)} [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau \]
\[ + \tilde{\eta}_j(y(t), u(t), t) \]

Theorem 3. A fault of class \( j \) is said to be isolable, if for each \( k \in \{1, \ldots, h\} \setminus \{j\} \) there exists a finite time \( t_k > 0 \) such that the following condition holds.
\[ \left| \mu |C_0| a_j e^{-\rho j(T+\theta)} \right| \int_{T_D}^{T+\theta} \mu |C_0| e^{\rho j(t)} [\tilde{\eta}_j(y(\tau), u(\tau), \tau)] d\tau \]
\[ + \tilde{\eta}_j(y(t), u(t), t) \geq 2 \overline{\varepsilon}_j(t) \] for \( t \geq T_D \). Therefore the occurrence of the \( j \)th class fault can be determined.

Remark 6. Compared with the analytical result in [12], the threshold (18) is empirical but less conservative. Using the method of [12], the bound of \( \left| C_0 \Omega_j(t) \tilde{f}_j(t) \right| \) would be \( \left| C_0 \Omega_j(t) \right| (|\tilde{f}_j(t)| - c_j + |r_j|) \), which is large and cannot be used to describe the residual decaying of the matched isolation observer.

Remark 7. This theorem presents the worst-case condition for fault isolation. Again, it is a sufficient condition. In most cases, a fault can be isolated without satisfying this strict condition. Based on this theorem, an upper bound of the isolation time of a \( j \)th class fault can be derived as \( T_D + \max\{ t_k \mid k \in \{1, \ldots, h\} \setminus \{j\} \} \).

V. SIMULATIONS

In this section, the effectiveness of the proposed fault detection and isolation method is illustrated with a simulated heating process.

This process consists of a water-water heat exchanger A, an air-water heat exchanger B with a cooling fan, long connected pipes, a three way valve, and some other elements. The schematic diagram of the plant is shown in Fig. 2, where the dash-dot line represents the primary (hot water) circuit and the dashed line represents the secondary (cold water) circuit. A simplified mathematical model of this system is [23].

![Fig. 2. Schematic of a heating process plant.](Image)
\[
\begin{align*}
\tau \dot{\theta}_e(t) &= (K_1 - 1)\left[\alpha(t - \tau_e)\dot{\theta}_e(t) + [1 - \alpha(t - \tau_e)]\dot{\theta}_e(t - \tau_e - \tau_a)\right] \\
&+ \dot{\theta}_e(t) + \dot{\theta}_e(t - \tau_e) - \dot{\theta}_e(t)(K_1 + 1) \\
\tau \dot{\theta}_o(t) &= K_2\left[\dot{\theta}_o(t - \tau_o) - \dot{\theta}_o(t) + \dot{\theta}_o(t - \tau_o)\right] \\
&+ \dot{\theta}_o(t) + \alpha(t - \tau_o)\dot{\theta}_0(t) + [1 - \alpha(t - \tau_o)]\dot{\theta}_0(t - \tau_o - \tau_a) \\
\tau \dot{\theta}_b(t) &= \left[\dot{\theta}_b(t - \tau_b) - \dot{\theta}_b(t) + K_1((u_f(t))(\dot{\theta}_b(t) - \dot{\theta}_o(t))\right]
\end{align*}
\]

(20)

where \( \dot{\theta}_d(t) \) is the outlet temperature of hot water of exchanger A, \( \dot{\theta}_o(t) \) and \( \dot{\theta}_b(t) \) are the outlet temperature of cold water of exchanger A and B, respectively. \( \theta_1(t) \) and \( \theta_0(t) \) are, respectively, the temperature of the heating water and the ambient temperature, which are assumed to be constant. \( \tau_e = 10s \) is the transportation delay between measuring point \( \dot{\theta}_d(t) \) and the three way valve. \( \tau_o = 7s \) is the delay between the three way valve and measuring point \( \theta_1(t) \). \( \tau_1 = 88s \) is the delay between measuring points \( \dot{\theta}_d(t) \) and \( \theta_1(t) \). \( \tau_a = 35s \) is the delay between measurement points \( \dot{\theta}_d(t) \) and \( \theta_0(t) \). Other parameters are \( \tau_e = 40s \), \( \tau_o = 20s \), \( \tau_1 = 100s \), \( K_1 = 3 \) and \( K_2 = 1.16 \). \( \alpha(t - \tau_e) \in [0, 1] \) is the opening in percentage of the three-way valve, which is the control input. \( K_3(u_f(t)) = 1.81 - 0.0089 \exp(1.112 u_f(t)) \) for \( u_f(t) \in [0, 4.5] \). \( \tau_f = 38s \) is the transportation delay between \( u_f(t) \) and \( \theta_0(t) \). \( \beta_a(t) = 0 = 41s \) is the transportation delay between the temperature of the heating water and \( \theta_0(t) \).

After linearizing nonlinear system (20) at the operating point of \( \theta_0 = 53.7^\circ C \), \( \theta_1 = 55.5^\circ C \), \( \theta_a = 39.9^\circ C \), \( \theta_f = 80^\circ C \), \( \theta_0 = 25^\circ C \), \( \alpha_0 = 0.23 \), and \( K_{30} = 1.05 \left( u_{f0} = 4V \right) \), we obtain the following linear system

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{3} A_i x(t - d_i) + B u(t) \\
&+ Z(\dot{Y}_e(t), u(t)) B(t) f + \eta_i(X_d(t), u(t), t) \\
y(t) &= C_0 x(t) + \eta_i(x(t), u(t), t)
\end{align*}
\]

where \( x(t) = [\theta_1(t) - \theta_0(t), \dot{\theta}_0(t) - \dot{\theta}_o(t), \theta_0(t) - \theta_30]^T \), \( u(t) = \alpha(t - \tau_e) - \alpha_0 d_1 = \tau_e + \tau_3 = 17s \), \( d_2 = \tau_c = 35s \), \( d_3 = \tau_1 = 88s \), and the parameter matrices are defined as follows

\[
A_0 = \begin{bmatrix}
(K_1 + 1)/\tau_1 & 1/\tau_1 & 0 \\
1/\tau_2 & (K_2 + 1)/\tau_2 & 0 \\
0 & 0 & -(K_{30} + 1)/\tau_3
\end{bmatrix},
\]

\[
A_1 = \begin{bmatrix}
(K_1 - 1)(1 - \alpha_0)/\tau_1 & 0 & 0 \\
(1 - \alpha_0)/\tau_2 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
0 & 0 & 1/\tau_1 \\
0 & 0 & (K_2 - 1)/\tau_2 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
A_3 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1/\tau_3
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
(K_1 - 1)(\theta_1 - \theta_{30})/	au_1 & \theta_1 - \theta_{30} & 0 \\
0 & (K_2 - 1)/\tau_2 & 0 \\
0 & 1/\tau_3 & 0
\end{bmatrix},
\]

\[
C_0 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

It is assumed that \( x(0) = [1, 0, 0]^T \) for \( \theta \in [-d_3, 0] \). The fault is represented as

\[
Z(Y_e(t), u(t)) B(t) f = \begin{bmatrix}
(K_1 - 1)u(t) + \alpha_0 \eta_1(t - d_1)/\tau_1 & 0 \\
(\alpha_0 - \alpha_0 \eta_1(t - d_1) + \alpha_0 - \theta_0)/\tau_2 & 0 \\
0 & 1/\tau_3
\end{bmatrix}
\]

\[
\eta_i(X_d(t), u(t), t) = \begin{bmatrix}
\eta_i(t)/\tau_1 \\
\eta_i(t)/\tau_2 \\
\eta_i(t)/\tau_3
\end{bmatrix}
\]

and the unknown disturbance on the state equation is the linearization error, which is described as

\[
\eta_i(x(t), u(t), t) = \begin{bmatrix}
\eta_i(t)/\tau_1 \\
\eta_i(t)/\tau_2 \\
\eta_i(t)/\tau_3
\end{bmatrix}
\]

where

\[
\eta_i(t) = (K_1 - 1) \eta_2(t),
\]

\[
\eta_2(t) = -(u(t) + \alpha_0) \eta_2(t - d_1) + \theta_{30} \\
- \alpha_0 \eta_2(t - d_1) - \alpha_0 \eta_1(t - d_1 - \theta_{30} u(t)) \\
- (u(t) + \alpha_0) \eta_2(t - d_1) - \eta_2(t - d_1) - \alpha_0 \eta_1(t - d_1),
\]

\[
\eta_3(t) = [K_3(u_f(t)) - K_3(u_f(t))](\theta_a - \theta_3(t) - \theta_{30}).
\]

The unknown disturbance on the output equation takes the form

\[
\eta_i(x(t), u(t), t) = \begin{bmatrix}
\eta_{i1}(t), \eta_{i2}(t)
\end{bmatrix}
\]

\[
[0.1 \sin(0.05 t), 0.1 \cos(0.05 t)]^T.
\]
We assume that \( u_f(t) = u_f[1 + 0.01\sin(0.05t + \pi/5)] \) and the control input \( u(t) \) is a square wave with period 1500s and magnitude ±0.05 \( \alpha_0 \).

Based on Lemma 3, the gain of the observers are selected as

\[
K_0 = K_1 = K_2 = \begin{bmatrix} 1.9101 & 0 \\ 0.1184 & 0 \\ 0 & 1.9799 \end{bmatrix}
\]

The parameters in the adaptive law (13) of the fault estimates are designed as \( \gamma_1 = 5, \gamma_2 = 5000, c_1 = 0.5, r_1 = 0.5, c_2 = 0, \) and \( r_2 = 10. \) From Lemma 1, it can be derived that \( \rho \leq 0.0573. \) So based on the analytical result of this lemma, the steady-mode gain of the right hand side of (8), \( |C_0|/\rho, \) is larger than 17.4520 (\( \mu \geq 1 \)), while the gain of the real system is much smaller as seen in the experiments. We select \( \rho_0 = \rho_1 = \rho_2 = 0.04. \) From the simulation, the following parameters can be determined: \( \mu_0 = \mu_1 = \mu_2 = 0.05, \alpha_0 = 1.11, \) and \( \alpha_1 = \alpha_2 = 1.25. \) Then, the detection and isolation thresholds are designed as

\[
\begin{align*}
\tau_0(t) &= 1.11 e^{-0.04t} + \int_0^t 0.05 e^{-0.04(t-\tau)} \bar{\eta}_1(\tau) \, d\tau, \\
\tau_i(t) &= \tau_i(t) = 1.25 e^{-0.04(t-\tau_D)} + \int_0^t 0.05 e^{-0.04(t-\tau)} \bar{\eta}_i(\tau) \, d\tau, \\
\text{where} \quad &
\bar{\eta}_1(\tau) = \sqrt{[(K_1 - 1)/\tau_1]^2 + (1/\tau_2)^2} \\
&\cdot \left[ (u(t) + \alpha_0) \psi_1(t - d_t) + \theta_0 \right] \\
&- \alpha_0 \theta_0 - \alpha_0 \psi_1(t - d_t) - \theta_0 u(t) \\
&+ c_0 \left[ \psi_1(t) + \theta_0 - \theta_0 \phi(t) \right] \\
&c_0 = \\
\max_{u_f(t) \in [0.99u_f, 1.01u_f]} \left[ (K_1(u_f(t)) - K_1(u_f[1 + 0.01\sin(0.05t + \pi/5)]))/\tau_1 \right]
\end{align*}
\]

**Experiment 1.** The gain of the actuator decreases to 0.9 at 2000s. This fault can be quickly detected at 2000.5s, when the residual of the detection observer surpasses \( \tau_0(t), \) as seen in Fig. 3. After this, two isolation observers are enabled to approximate the fault. At the time of 2116.0s, the residual of the 2nd isolation observer surpasses its threshold, while the residual of the 1st isolation observer is below its threshold. So at this time, conclusion can be drawn that a fault of class 1, i.e., the actuator gain fault, has occurred. The fault mismatch function surpasses \( 2\tau_i(t) \) at 2149.3s, showing that this fault satisfies the isolability condition of Theorem 3. The fault estimate of the 1st isolation observer is clearly better than that of the 2nd isolation observer.

**Experiment 2.** An additive fault with magnitude −5 on the third state equation occurs at 2000s. From Fig. 4, it can be observed that this fault is also detected quickly at 2000.3s and isolated at 2108.2s when the residual of the 1st isola-
tion observer surpasses its threshold. This fault also satisfies the isolability condition in Theorem 3, as the fault mismatch function surpasses \( 2\epsilon_0(t) \) at 2131.1s. It is obvious that in this experiment the fault estimate of the 2nd isolation observer is better than that of the 1st isolation observer.

\[\begin{align*}
\text{VI. CONCLUSION} \\
A robust fault detection and isolation scheme has been developed for linear continuous retarded systems with unstructured faults and disturbances by combing an adaptive observer design with a multi-observer strategy. The determination of adaptive thresholds is realized and illustrated via theoretical analysis and experiments. The sensitivity of the detection algorithm to fault, estimation performance of the isolation observers, the fault isolability condition are all analyzed theoretically in the paper. Simulation applications to a heating process show that the proposed scheme is effective.
\end{align*}\]

\[\text{APPENDIX} \]

\[\text{PROOF OF LEMMA 2} \]

\[\text{Proof.} \] Considering that each column of \( \Psi(t) \) is a solution of Eq. (3) for \( t \geq 0 \) [18], we can derive that \( \Psi(t|l) \) for all \( l \in \mathcal{W} = \{ w \in \mathbb{R}^n \mid |w| = 1 \} \) are also solutions. Then, there exist \( \mu, \rho > 0 \) from Lemma 1 and a \( l' \in \mathcal{W} \), such that \( |\Psi(t|l)| = \sup_{t \geq 0} |\Psi(t|l)| \leq \mu e^{\rho|l|} \) for \( t \geq 0 \). From \( \Psi(0) = I_n \) and \( \Psi(0) = 0_n \), we can derive \( \|\Psi(t|l')\| = |l'| = 1 \), and \( |\Psi(t)| \leq \mu e^{\rho|l|} \) for \( t \geq 0 \). Furthermore, since the solution of Eq. (1) is the summation of the solution under zero input condition and that under zero initial condition, we can derive estimate (4).

\[\Box\]

\[\text{REFERENCES} \]


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