DEVELOPMENT OF CVT CONTROL SYSTEM AND ITS USE FOR FUEL-EFFICIENT OPERATION OF ENGINE

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ABSTRACT

Continuously variable transmission (CVT) provides an automobile with the ability to change the gear ratio continuously, which can then improve not only ride quality such as acceleration performance but also fuel-efficiency. However, to take advantage of the ability, a control system that can precisely control the gear ratio is required. This paper proposes such a control system for a belt-driven CVT system. For controller design, first the CVT system is modeled by analytical and experimental approaches. The resultant static and dynamic characteristics provide a nonlinear first-order model with an uncertain time constant and time delay. The nonlinear steady-state gain is adjusted to one by a gain-scheduled pre-compensator. Thereby the plant model becomes a linear first-order lag system with a dead time. The next step is controller design using the plant model. To guarantee stability and control performance against the parameter variation and time delay, the $\mu$-synthesis, a robust control method, is employed for feedback control. In addition, a feedforward controller is incorporated into the feedback control system to obtain better output response. The feedforward controller is given by a combination of the inverse system of the plant and a reference model that gives desired output response. As a result, the control system becomes a two-degree-of-freedom control system. To evaluate the performance of the control system and its effectiveness on the fuel-efficiency, computer simulation and driving tests were conducted. The simulation and experiment results prove that the proposed control system can make the gear ratio track a reference output quickly and precisely in the presence of the uncertainties. The results also show that the control system improves fuel-efficiency by changing the gear ratio so that the engine torque and its revolution speed can satisfy optimum-efficiency operating condition.

KeyWords: Automotive, robust control, two-degree-of-freedom control, CVT, fuel-efficiency.

I. INTRODUCTION

Acceleration performance of vehicles is often mentioned as one of the most important factors for customers to choose their cars. On the other hand, improvement in fuel-efficiency of an engine is strongly required for not only an economic reason but also an environmental reason. However, improving both acceleration performance and fuel-efficiency at the same time is not a simple problem,
and it is largely studied around the world. Fuel efficiency of an engine generally differs depending on its running condition, such as rotation speed, amount of fuel exhaust, and amount of air, even if its output power is the same. This means that there is a possibility that we can achieve good fuel efficiency and good acceleration characteristics at the same time if we can drive the engine at its optimum efficiency point for a required power. As Fig. 1 shows, an optimum efficiency point is determined as a point on specific fuel consumption map indicated by the dashed line for positive torque or a point on the fuel cutoff line for negative torque; namely, a particular combination of function of engine revolution speed and torque provides an optimum point. Since the gear ratio affects those variables, operation at the optimum efficiency points is actually possible by changing the gear ratio appropriately. In addition, we can avoid losing too much energy if we can adjust the gear ratio properly to make this fuel-cutoff time longer, because engines exhaust no fuel in deceleration state when the drivers release the acceleration pedal which is called coasting. Therefore, to track optimum efficiency operating points, we need a continuously variable transmission (CVT) system with an accurate gear ratio control system.

In this paper, we present a design method of a gear ratio control system for a CVT system as well as its modeling. The CVT system that we adopted is a belt-driven one, where the gear ratio continuously changes by adjusting the hydraulic pressure force on the pulleys. Figure 2 shows a power-train system with the CVT mechanism. This belt-driven CVT system has non-linear properties such as a delay time constant that varies with oil pressure force applied on the pulleys. The CVT system also has another uncertainty, time delay. In order to deal with the uncertainties, we have modeled the CVT system as a first-order lag system with an uncertain time constant and time delay, and applied the µ-synthesis, a robust control method, to controller design. Seeking for sufficient robustness by the feedback control generally makes control performance insufficient. However, the control performance can be improved by combining a feedforward controller with the feedback control system. The control system of this type is called a two-degree-of-freedom control system, which can achieve desirable robustness and control performance at the same time.

To evaluate the performance of the control system and its effectiveness on the fuel efficiency, we have conducted computer simulation and driving tests. One is the test how precisely the optimum fuel efficient points can be traced and the other is the one how long a good rotation speed can be kept in fuel-cutoff condition in the coasting.

The rest of the paper is organized as follows. Modeling of the CVT system is described in Section 2 and controller design is presented in Section 3. In Section 4 and Section 5, simulation results and experimental results are shown respectively. Finally conclusions are given in Section 6.

II. CONTINUOUSLY VARIABLE TRANSMISSION PLANT CHARACTERISTICS

This CVT has a stepper motor that drives a servo valve of the hydraulic system, which adjusts the pulley clamping force to obtain the required gear ratio. Hence, we choose the stepper motor position as an input and the gear ratio as an output. In designing a new gear ratio servo system, it is necessary to have a clear understanding of the static and dynamic transmission characteristics. For that purpose, we analytically and experimentally investigated the relation between the input and output.

2.1. Static characteristics

In the CVT system, the stepper motor position changes the pulley spacing, which determines the gear ratio through mechanical linkage shown in Fig. 3. From the geometry of the transmission mechanism, we can derive the steady-state gear ratio $i_p$ as a function of the amount of change in the pulley spacing as follows.
Fig. 3. Construction of continuously variable transmission system.

\[ r_p = \frac{D_S}{2 \tan(\beta)} + r_{p\text{min}} \]  \hspace{1cm} (1)

\[ r_S = \frac{2r_p - \pi D_C + \sqrt{a_1^2 - 4a_2}}{2} \]  \hspace{1cm} (2)

\[ i_p = \frac{r_S}{r_p} \]  \hspace{1cm} (3)

where

\[ a_1 = 2r_p - \pi D_C \]  \hspace{1cm} (4)

\[ a_2 = r_p^2 + \pi D_C \]  \hspace{1cm} (5)

Figure 4 shows the steady-state pulley ratio calculated with these equations corresponds well with the experimental data.

2.2. Dynamic characteristics

The hydraulic pressure on the primary pulley changes the gear ratio along with the line pressure that sustains the secondary pulley. The line pressure, which is used to generate and maintain a suitable level of belt clamping force, widely varies depending on the driving conditions, and it has a significant effect on the dynamic characteristics of the plant. The effect was investigated through experiments of the pulley ratio step response under the conditions of the maximum and minimum line pressures during upshifting and downshifting. The step responses in Fig. 5 suggest that the transfer function of the plant can approximately be modeled as a first-order lag system with a dead time. Moreover, as shown in Fig. 6, the first-order lag time constant of the plant changes with the gear ratio \( i_p \) (or stepper motor position \( \theta \)), shift direction \( s_D \) and line pressure \( p_L \), and the steady-state gain \( K_p \) varies with \( \theta \); however, the plant dead time \( T_p \) is constant, unaffected by the variables or parameters. Thus, we can describe the plant dynamics as

\[ G_p(s) = \frac{i_p(s)}{\theta(s)} = \frac{K_p(0)}{T_p(i_p, s_D, p_L)s + 1} e^{-i_p s} \]  \hspace{1cm} (6)

where \( s \) is a differential operator.
III. DESIGN OF GEAR RATIO SERVO SYSTEM

We impose the following requirements on the gear ratio servo system.
1. There must be no steady-state error between the target gear ratio and the actual gear ratio.
2. The servo system must have the required steady-state and transient characteristics.
3. Gear ratio response must be relatively unaffected by external disturbances and/or parameter variation.

As mentioned earlier, the step response of the gear ratio differs depending on the line pressure. In addition, the relationship between the stepper motor position and the gear ratio may vary somewhat from one CVT unit to another on account of normal variation that occurs during manufacturing.

To meet the requirements and system uncertainties, we design a two-degree-of-freedom control system that has feedback and feedforward controllers. The feedback controller provides robustness against the uncertainties and the feedforward controller improves output response characteristics independent of the feedback characteristics. We employ μ-synthesis, which is one of the robust control techniques, to design the feedback controller.

In the following we describe modeling of the plant and design of the controllers in detail.

3.1. Gain-compensated plant model

Before applying the linear control, we should compensate for known nonlinear characteristics of the plant by nonlinear compensation to help the linear control work well. We apply gain compensation to eliminate the dependence of steady-state gain on the stepper motor position. Multiplying the plant model by a map allows us to make the steady-state gain of the plant model of Eq. (7), regarding the transfer function of the plant modeled dynamics.

\[ \phi(s) = \frac{1}{T_p(i_P, s_G, p)} + 1 \]  

The plant modeled by Eq. (7) can be written as

\[ i_p = \frac{1}{T_p}(-i_p + i_{PS} + w_p) \]

where

\[ w_p = \frac{\Delta T_p \Delta_i}{T_p} i_p \]

\( \Delta_i \) is a real number that satisfies \( |\Delta_i| < 1 \). By considering \( w_p \) as an external input, we can treat the time constant as real valued perturbation.

Next, we consider the dead time of the transfer function of the CVT system in Eq. (6). In particular, the dead time \( L_p \) tends to increase due to aging. By modeling the unmodeled dynamics as a multiplicative uncertainty, we can describe it as

\[ W_{L0}(s) = e^{-2sL_p} - 1 \]  

where the dead time is considered to be twice as large as the nominal dead time \( L_p \). However, since the transfer function of Eq. (11) is not a finite-order linear time-invariant system, it is not appropriate as a weighting function. Therefore, we replace \( W_{L0}(s) \) by the weighting function in Eq. (12) whose gain of the frequency response is slightly larger than that of \( W_{L0}(s) \).

\[ W_L(s) = \frac{4.2L_p s}{2L_p s + 1} \]

3.2. Design of feedback controller

Since \( G_p(s) \) is a linear parameter varying (LPV) system, we could apply gain scheduling control to design of the feedback controller. However, we must also take into account the uncertainties due to the line pressure and time delay. This is the reason that we employ the μ-synthesis for the plant model of Eq. (7), regarding \( T_p \) as a constant with a certain variation. Figure 7 shows a block diagram of the control system.

A variation range of \( T_p \) was determined from step response experiments conducted under various operating conditions that depend on the values of \( i_p, s_G, \) and \( p \). The maximum and minimum values of the time constant in gear ratio control are denoted by \( T_{p_{\max}} \) and \( T_{p_{\min}} \), respectively.

The nominal time constant \( T_p \) is defined as the average of the maximum and minimum values, i.e.,

\[ T_p = \frac{T_{p_{\max}} + T_{p_{\min}}}{2} \]

\[ \Delta T_p = |T_{p_{\max}} - T_p| = |T_{p_{\min}} - T_p| \]

The above gain compensation and simplification modify the transfer function of the plant \( G_{p}(s) \) in Eq. (6) as

\[ i_p = \frac{1}{T_p}(-i_p + \phi(s)) + w_p \]  

\[ w_p = \frac{\Delta T_p \Delta_i}{T_p} i_p \]

\( \Delta_i \) is a real number that satisfies \( |\Delta_i| < 1 \). By considering \( w_p \) as an external input, we can treat the time constant as real valued perturbation.
Figure 8 shows a gain diagram of the transfer function in Eqs. (11) and (12). It is seen from this figure that the modeling error of the gear ratio modeled as a multiplicative uncertainty does not exceed the magnitude of $W_f(s)$.

To evaluate control performance, the criterion output of the system is chosen as

$$z(s) = W_f(s) W_e(s) e(s)$$  \hspace{1cm} (13)

where $e = r - i_p$ is control error, $W_e$ is a constant, and $W_f(s)$ is given by Eq. (14).

$$W_f(s) = \frac{s+1}{s}$$  \hspace{1cm} (14)

$W_f(s)$ is given an integral characteristic to eliminate steady-state error for the constant command and input disturbance [8]. This is known as the internal model principle. The generalized plant is shown in Fig. 9, where $W_f(s)$ is placed in the feedback loop. Letting $K'(s)$ denote the controller obtained for the generalized plant, we multiply $K'(s)$ by $W_f(s)$ to obtain the final controller, since $W_f(s)$ is in the feedback path. Thus, the feedback controller $K(s)$ is given by

$$K(s) = W_f(s) K'(s)$$  \hspace{1cm} (15)

3.3. Design of feedforward controller

To improve command-tracking performance, we incorporate feedforward control into the control system. Figure 7 shows a block diagram of the two-degree-of-freedom control system. The feedforward path including $G_{sf}(s)$ carries out the feedforward compensation. $G_{sf}(s)$ is designed as follows. First we choose a reference model $G_{isf}(s) e^{s^2p_s}$. $G_M(s)$ is a first-order lag system $1/(T_{p} s + 1)$ multiplied by $e^{s^2p_s}$. The time delay is added, since the nominal plant has a dead time $L_p$. We compose the feedforward controller $G_{sf}(s)$ of the reference model $G_{sf}(s)$ and the inverse system of the plant $G_{ps}(s)$ as

$$M_M(s) = \frac{G_M(s)}{G_{ps}(s)} = \frac{T_P(i_p, s_{D}) + 1}{T_p s + 1}$$  \hspace{1cm} (16)

where the delay time constant $T_P$ is scheduled as a linear function of the gear ratio $i_p$ and the shift direction $s_{D}$ to alleviate the effect of the parameter variation; that is, a map of the time constant $T_P$ with respect to $i_p$ and $s_{D}$ is made and $T_P$ is updated by table lookup. Note that although actually $T_P$ is also a function of $p_s$ as in Eq. (6) or (7), it is ignored here to avoid complicating the controller design; instead we regard it as an uncertainty in the feedback controller design. The feedforward controller matches an open-loop transfer function from the command input $i_{pc}$ to the output $i_p$ to the reference model for the nominal plant. Therefore, the feedforward control provides a desired time response of $i_p$. However, the modeling error between the actual plant and the nominal plant degrades the control performance. The feedback control compensates for the degradation and guarantees the closed-loop stability.

IV. SIMULATION RESULTS

We applied the design method to a test vehicle. In the $\mu$-synthesis, the variation of the plant delay time constant $T_p$ was set to 80% or 1.8$T_{p0}$ and the dead time to twice the nominal value $L_p$. Originally 16th order controller was obtained by the $\mu$-synthesis; however, it was reduced to a third-order one by approximate pole-zero cancellation and balanced truncation. The $\mu$-analysis proved that the resulting controller achieved a peak $\mu$-value of 0.99 as Fig. 10 shows, which means that the specified robust stability and control performance are satisfied.

To evaluate the performance of the gear ratio control system, computer simulation was conducted using a precise nonlinear model. The feedback controller was discretized by Tustin’s method with a sampler and a zero-order hold. The reference model and the feedforward controller were also discretized by backward Euler’s method, where a sampling period of 10 milliseconds was chosen.
We chose the case of kick-down shift, which is a demanding operation where $T_p$ can become large as Fig. 6 shows, and we consider two simulation conditions. One is the nominal condition where $T_p = T_p(i_p, s_D, p_L)$ and the dead time is $L_p$. The other is a worst case where the plant has the estimated largest time constant $T_p = 1.8T_p(i_p, s_D, p_L)$ and the presumed largest dead time $2L_p$.

Figure 11(a) shows the simulation results for the nominal condition. The actual gear ratio response coincides well with the reference output except for the short period immediately after the step input is applied, where the gear ratio changes fast. Figure 11(b) shows the simulation results for the worst condition. Although the control performance shows some degree of deterioration compared with the nominal case, such as increased deviation from the reference gear ratio, the control system remains stable even in such an adverse condition. The results of the simulations conducted in other operations and plant conditions also proved that this gear ratio control system satisfied the requirements for stability and control performance.

V. DRIVING TEST

Having obtained the good simulation results, we installed the gear ratio servo system in an actual vehicle to verify its performance in a driving test. The driving test was executed for the rectangular acceleration pedal command, which generated a fast gear ratio command, to evaluate the control performance. Also, we evaluated the performance to trace the optimum fuel-efficiency line of the engine through the test of how long good rotation speed could be kept in the fuel-cutoff condition in the coasting.

Figure 12 shows the experimental results. In Fig. 12(a), time histories of the acceleration pedal opening angle, longitudinal acceleration, engine revolution speed, CVT gear ratio and vehicle velocity. The actual gear ratio nearly coincides with the command determined so that the engine torque and revolution speed can become optimum in fuel efficiency. Figure 12(b) shows engine revolution speed-versus-engine torque plots during the test. The upper dashed line indicates the optimum fuel efficiency operating condition and the lower one the engine fuel cutoff condition. Although small overshoot appears, especially in the transient response at the pedal-on and -off actions at 5s and 15s, respectively, since the engine response is faster than the gear ratio response, the desired operating conditions are maintained in the steady state. Namely, the engine is driven in its optimum fuel efficiency point in the acceleration, and it keeps a good rotation speed in the coasting. Thus, the power train system with the CVT achieves both optimum fuel efficiency and desired acceleration response at the same time.
VI. CONCLUSION

We have presented modeling of a belt-driven CVT system and design of its controller. Analytical and experimental investigation reveals that the plant dynamics can be described by a first-order lag system with an uncertain time constant, nonlinear steady-state gain and time delay. The gain-scheduled nonlinear pre-compensation and the robust control method are employed to deal with the nonlinearity and uncertainties. Although the feedback controller provides desirable stability and robustness, the feedforward controller is combined, since the response speed is insufficient with the feedback control only. Thus, the CVT control system becomes a two-degree-of-freedom control system. The simulation and experiment results indicate that the control system satisfies specified robust stability and control performance in the presence of the uncertain parameter variation and unmodeled dynamics. The results also suggest that the modeling of the plant and its uncertainties is successful. The driving tests prove that the ability of the controller to change the gear ratio fast and accurately makes it possible to trace the optimum fuel-efficiency line, so that simultaneous pursuit of desired fuel-efficiency and acceleration characteristics is achieved.

NOMENCLATURE

\( G_P(s) \)  transfer function of plant
\( G_{PS}(s) \) simple modification of transfer function of plant
\( G_{pu}(s) \) transfer function of plant for feedback controller design
\( G_M(s) \) first-order lag system of reference model part
\( W_L(s) \) weighting function for feedback controller
\( W_E(s) \) evaluation function for feedback controller
\( W_I(s) \) transfer function of internal model principle for feedback controller with integral-characteristics
\( T_P \) first-order lag time constant of plant
\( T_{Pn} \) nominal value of time constant of plant
\( T_{Pmax} \) maximum value of time constant of plant
\( T_{Pmin} \) minimum value of time constant of plant
\( \Delta T_P \) maximum variation of time constant of plant
\( K_P \) steady-state gain of plant
\( L_P \) dead time of plant
\( T_R \) first-order lag time constant of reference transfer function of gear ratio servo system
\( i_P \) gear ratio
\( i_{PS} \) output of gear ratio control system, as shown in Fig. 7
\( s_D \) shift direction
\( r_P \) primary pulley radius
\( r_S \) secondary pulley radius
\( r_{Pmin} \) minimum radius of primary pulley
\( \beta \) sheave angle of primary pulley
\( D_C \) distance between primary and secondary pulleys
\( D_S \) change in spacing between pulleys
\( L_B \) belt length
\( s \) differential operator
\( \theta \) stepper motor position
\( r \) equation of the \( i_{PR} \) (: reference gear ratio)

REFERENCES

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