CONTROL LAW FOR QUADRATIC STABILIZATION OF PERTURBED FUZZY TIME-DELAY LARGE-SCALE SYSTEMS VIA LMI

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ABSTRACT

In this paper, the perturbed continuous-time large-scale system with time delays is represented by an equivalent Takagi-Sugeno type fuzzy model. First, two types of decentralized state feedback controllers are considered in this paper. Based on the Riccati-type inequality, the Razumikhin theorem, and the delay-dependent Lyapunov functional approach, some controller design approaches are proposed to stabilize the whole fuzzy time-delay system asymptotically. In these design methods, both the delay-independent and delay-dependent stabilization criteria are derived. By Schur complement, these sufficient conditions can be easily transformed into the problem of LMI’s. Moreover, the systems with all the time-delays \( \tau_{ij}(t) \) are the same for all rules (i.e., \( \tau_{ij}(t) = \tau_{ij}(t) = \tau_{ij} \) for all \( l \neq m \)); the authors also propose a simpler and less conservative stabilizing criteria. A numerical example is given to illustrate the control design and its effectiveness.

KeyWords: Fuzzy large-scale system, delay-dependent stabilization criteria, time delay, LMI.

I. INTRODUCTION

Large-scale systems can be found in many real-life practical applications such as electric power systems, nuclear reactors, aerospace systems, economic systems, process control systems, computer networks, and urban traffic network, etc. Time-delays, though, are frequently a source of instability and are often encountered in various engineering systems. For the past decade and even before, many researchers have paid a great deal of attention to various control methods in the stabilization, estimation and robustness of large-scale time-delay systems, such as [1-3].

However, the above papers only deal with the problems of conventional control systems. Recently, the task of effectively controlling nonlinear and large-scale systems described by T-S fuzzy model has been also noticed. The design methods of quadratic stabilization of uncertain fuzzy systems were proposed by using the state feedback control [4-6]. A \( H_{\infty} \) decentralized fuzzy model reference tracking control design method for nonlinear large-scale systems has been proposed by [7]. Hsiao and Hwang [8] is concerned with the stability problem of fuzzy large-scale systems. Wang et al. [9] proposes a fuzzy controller design method to stabilize a large-scale fuzzy system. The problem of robust stabilization of perturbed discrete time-delay large-scale systems is considered in [10]. The paper [11] proposed some delay-dependent stabilization criteria for T-S fuzzy systems with time delays. The stability and stabilization criteria were derived for T-S fuzzy large-scale systems in [12]. For T-S fuzzy interconnected systems with multiple time delays, the paper [13] proposed the stability analysis method.

Some new approaches for decentralized stabilization of time-delay fuzzy large-scale systems with nonlinear perturbations are developed in this paper. First, the researchers design a decentralized state feedback fuzzy controller to stabilize this system robustly. Next, a conventional decentralized state feedback controller is proposed to solve the same problem as above. In these design ap-
proaches, one can determine the controller gains using LMI's tool. At the same time, the perturbation bounds are obtained. Moreover, the delay-dependent criteria are also derived for time-delay fuzzy large-scale systems without perturbations. Finally, an example is proposed to illustrate the stabilization task.

II. SYSTEM DESCRIPTION

Consider a perturbed time-delay large-scale system $S$ composed of $N$ subsystems $S_i$, $i = 1, 2, ..., N$. Each rule of the subsystem $S_i$ can be represented by a T-S fuzzy model as follows: [7,9]

$$
S_i^j : \begin{cases}
\text{If } z_{i1} = F_{i1}^j \text{ and } \ldots \text{ and } z_{in} = F_{in}^j,
\text{Then } \dot{x}_i(t) = A_i^j x_i(t) + B_i^j u_i(t)
\end{cases},
$$

(1)

$$
S_i^j : \begin{cases}
\text{If } z_{i1} = F_{i1}^j \text{ and } \ldots \text{ and } z_{in} = F_{in}^j,
\text{Then } \dot{x}_i(t) = A_i^j x_i(t) + B_i^j u_i(t)
\end{cases} + \sum_{j=1}^{N} \left[ A_i^j x_i(t - \tau_i^j(t)) + f_i^j(x_i(t), t) \right]
$$

(1)

$$
i = 1, 2, ..., N; l = 1, 2, ..., n, \text{ where } x_i(t) \in \mathbb{R}^n \text{ and } u_i(t) \in \mathbb{R}^m \text{ are the state and control vectors of the } i\text{th subsystem, respectively. } F_{i1}^j (q = 1, 2, ..., n) \text{ and } r_i \text{ represent the linguistic fuzzy variables of the rule } l, \text{ and the number of the fuzzy rules in subsystem } S_i, \text{ respectively;} \text{ and } z_i(t) = [z_{i1}, z_{i2}, \ldots, z_{in}] \text{ are the premise variables for subsystem } S_i. A_i^j \text{ and } B_i^j \text{ denote the system matrix and input matrix with appropriate dimensions, respectively. } \tau_i^j \in \mathbb{R} \text{ and } A_i^j \in \mathbb{R}^{n \times n}, \text{ represent the time delay and the interconnection matrix between the } i\text{th and the } j\text{th subsystems. } x_i(t) = \psi_i(t), t \in [-\tau, 0], \text{ and } \psi_i(t) \text{ is the initial condition of the state. The vector } f_i^j(x_i(t), t) \text{ is a nonlinear perturbation. All the subsystems satisfy the following assumption.}

(A1) For each rule $f_i^j(x_i(t), t)$, there exists a constant $b_i^j > 0$ such that $\| f_i^j(x_i(t), t) \| \leq b_i^j \| x_i(t) \| [5,9]$. If we utilize the standard fuzzy inference method, i.e., a singleton fuzzifier, minimum fuzzy inference, and central-average defuzzifier, (1) can be inferred as

$$
\dot{x}_i(t) = \sum_{j=1}^{N} h_i^j(z_i(t)) \left[ A_i^j x_i(t) + B_i^j u_i(t) \right] + \sum_{j=1}^{N} \left[ A_i^j x_i(t - \tau_i^j(t)) + f_i^j(x_i(t), t) \right]
$$

(2)

where

$$
h_i^j(z_i(t)) = \frac{w_i^j(z_i(t))}{\sum_{j=1}^{N} w_i^j(z_i(t))},
$$

(3)

$$
F_{i1}^j(z_i(t)) \text{ is the grade of membership of } z_i(t) \text{ in } F_{i1}^j. \text{ It is seen that } w_i^j(z_i(t)) \geq 0, i = 1, 2, \ldots, r, \text{ for all } t, \text{ and } \sum_{j=1}^{N} h_i^j(z_i(t)) = 1.
$$

III. DELAY-INDEPENDENT STABILIZATION OF PERTURBED FUZZY TIME-DELAY LARGE-SCALE SYSTEMS

In this section, the decentralized control scheme and the fuzzy control approach are employed to design the state feedback controllers. First, let the $i$th fuzzy controller (type-I) corresponding to $S_i$ be as the form:

$$
C - R_i^j \sim I : \begin{cases}
\text{If } z_{i1} = F_{i1}^j \text{ and } \ldots \text{ and } z_{in} = F_{in}^j,
\text{Then } u_i(t) = -K_i^j x_i(t).
\end{cases}
$$

(4)

$$
i = 1, 2, ..., N; l = 1, 2, ..., n. \text{ For convenience, the authors use the briefness notation } h_l^i \text{ to denote } h_l(z_i(t)) \text{ in (3). Analogous to (2), the final output of the fuzzy controller for each subsystem } S_i \text{ is}

$$
\dot{x}_i(t) = \sum_{j=1}^{N} \sum_{l=1}^{N} h_{i}^{j} h_{l}^{n} \left[ G_{i}^{m} x_i(t) + \sum_{j=1}^{N} \left[ A_i^j x_i(t - \tau_i^j(t)) + f_i^j(x_i(t), t) \right] \right]
$$

(6)

in which $G_{i}^{m} = A_i^j - B_i^j K_i^m$.

Second, the authors consider other controllers (type-II) corresponding to $S_i$ as follows: (i.e., the conventional decentralized state feedback controller)

$$
u_i(t) = -\sum_{j=1}^{N} K_i^j x_i(t).
$$

(7)

Thus, combining (2) and (7), the closed-loop fuzzy subsystem becomes

$$
\dot{x}_i(t) = \sum_{j=1}^{N} \sum_{l=1}^{N} h_{i}^{j} h_{l}^{n} \left[ G_{i}^{m} x_i(t) + \sum_{j=1}^{N} \left[ A_i^j x_i(t - \tau_i^j(t)) + f_i^j(x_i(t), t) \right] \right]
$$

(8)
Lemma 1. [2] For any two matrices $X$ and $Y$ with appropriate dimension, one has
\[ X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y, \]
for any constant $\varepsilon > 0$.

Lemma 2. [14] Tchebyshev’s inequality holds for any matrix $v_i \in \mathbb{R}^{m \times n}$
\[ \left[ \sum_{i=1}^{n} v_i \right]^T \left[ \sum_{i=1}^{n} v_i \right] \leq m \left[ \sum_{i=1}^{n} v_i \right]^T v_i \] (10)

Lemma 3. [11] For any two matrices $X_i$ and $Y_i$ for $i = 1, 2, \ldots, r$, and $\varepsilon > 0$ with appropriate dimensions, one has
\[ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{ij} X_i^T S Y_j \leq \sum_{i=1}^{r} \mu_i (X_i^T S X_i + Y_i^T S Y_i), \] (11a)
\[ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} \mu_{ij} \mu_{kl} X_{il}^T S Y_{lj} \leq \sum_{i=1}^{r} \mu_i (X_i^T S X_i + Y_i^T S Y_i) \] (11b)

Lemma 4. [8] Assume that $v_1(\cdot) \in \mathbb{R}^r$, $v_2(\cdot) \in \mathbb{R}^s$, and $\Theta \in \mathbb{R}^{m \times n}$ are defined on the interval $\Omega$. Then, for any matrices $\Phi \in \mathbb{R}^{m \times n}$, $\Psi \in \mathbb{R}^{s \times m}$, and $\Xi \in \mathbb{R}^{m \times s}$, the following holds:
\[ -2 \int_{\Omega} \left( v_1(\Theta) \Phi v_2(\Theta) \right) d\theta \]
\[ \leq \left[ \begin{array}{cc} v_1(\Theta) & v_2(\Theta) \end{array} \right]^T \left( \begin{array}{cc} \Phi & \Psi \end{array} \right) \Xi \left( \begin{array}{c} v_1(\Theta) \\ v_2(\Theta) \end{array} \right), \] (12)
where $\left[ \begin{array}{cc} \Phi & \Psi \end{array} \right] \Xi \geq 0$.

Lemma 5. [11] For any two matrices $X$ and $Y = Y^T > 0$ with appropriate dimensions, one has
\[ X^T Y^{-1} X > X^T + X - Y \] (13)
Now, the first theorem can be proposed. (Type-I controller)

Theorem 1. Consider the fuzzy time-delay large-scale system $S$ as (1). Suppose the assumption A1) hold and the state feedback gain is $K_i = \rho_i B_i^T P_i$, where $P_i^T = P_i > 0$ is the solution of (14) as following
\[ A_i^T P_i + P_i A_i - P_i B_i B_i^T P_i + Q_i \leq 0 \]
\[ i = 1, 2, \ldots, N, \] (14)
where $Q_i = Q_i^T > 0$. Then the overall closed-loop fuzzy time-delay system composed of $N$ subsystems (6) is stabilized asymptotically by the type-I fuzzy controller (4) if there exist $P_i, \rho_i, Q_i$, and $U_i > 0$ satisfying the following conditions, respectively.
\[ \left\{ \begin{array}{l} P_i > U_i^{-1} \\ \Omega_i = \Omega_i^T + (2\rho_i - 1)P_i B_i B_i^T P_i - P_i - P_i \left( \sum_{j=1}^{N} A_{ij} U_{ij} A_{ij}^T \right) P_i \geq 0 \\ \Xi_i = \Xi_i^T - P_i B_i B_i^T P_i - P_i \left( \sum_{j=1}^{N} A_{ij}^T U_{ij} A_{ij} \right) P_i \geq 0 \\ \Theta_i = \Theta_i^T - P_i B_i B_i^T P_i - P_i \left( \sum_{j=1}^{N} A_{ij}^T U_{ij} A_{ij} \right) P_i \geq 0 \end{array} \right. \] for all $l = m$ (15a)
\[ N \left( h_i^2 + \lambda_{\max}(P_i) \right) \leq \Lambda_i \] (15b)

where $\Lambda_i = \min \left( \Gamma_i^1, \Gamma_i^2, \Gamma_i^3 \right)$, and $h_i = \max \left( h_{ij} \right)$, for $i, j = 1, 2, \ldots, N, \Delta l, m = 1, 2, \ldots, r_i$ respectively.

Proof. Let the Lyapunov function for the closed-loop system (6) be as follows:
\[ V(x) = \sum_{i=1}^{N} x_i^T P_i x_i = \sum_{i=1}^{N} v_i(x_i), \quad i = 1, 2, \ldots, N. \] (16)
where $P_i^T = P_i > 0$ satisfies (14). Then, the time derivative of $V(x)$ along the trajectory of system (1) with state feedback (4) is
\[ V = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + x_i^T P_i X_i \right\} \]
\[ = \sum_{i=1}^{N} \left( h_i^2 \right) \left( x_i^T \left( G_i^T P_i + P_i G_i \right) x_i \right) + 2 x_i \sum_{j=1}^{N} \left\{ P_i A_{ij} x_j \left( t - \tau^j_0(t) \right) + P_i f_{ij}(x_j) \right\} \] (17)
Using Lemma 1, 2, the inequality (14), assumption A1), and let \( P_i \geq U_i^{-1} \) for all \( i, j, l \), then we have

\[
V \leq \sum_{j=1}^{N} \left( \sum_{l=1}^{r} (h_{ij})^2 \right) \left| -x_i^T Q_j x_j - (2p_i^l - 1) x_i^T P_i B_i^T P_i x_i \right| + x_i^T P_i x_i + x_j^T P_j x_j
\]

\[
+ \sum_{l=1}^{r} \left( \sum_{j=1}^{N} (h_{ij})^2 \right) \left| V_j \left( x_j \left( t - \tau_j^l(t) \right) \right) + x_j^T b_j^T x_j \right|
\]

\[
+ \sum_{j=1}^{N} \left( \sum_{l=1}^{r} (h_{ij})^2 \right) \left| V_i \left( x_i \left( t - \tau_i^l(t) \right) \right) + x_i^T b_i^T x_i \right|
\]

\[
+ \sum_{j=1}^{N} \left( \sum_{l=1}^{r} (h_{ij})^2 \right) \left| V_i \left( x_i \left( t - \tau_i^l(t) \right) \right) + x_i^T b_i^T x_i \right|
\]

\[
+ \sum_{j=1}^{N} \left( \sum_{l=1}^{r} (h_{ij})^2 \right) \left| V_j \left( x_j \left( t - \tau_j^l(t) \right) \right) + x_j^T b_j^T x_j \right|
\]

(18)

If \( \Omega^l_j > 0, \Xi^l_j > 0, \) and \( \Theta_i^m > 0 \), then we have

\[
\dot{V} < \sum_{i=1}^{N} \left[ \left| -x_i^T A_{i1} x_i + x_i^T d_{Nj} P_i x_i + x_i^T b_i^T x_i \right| + \sum_{j=1}^{N} \left( \sum_{l=1}^{r} (h_{ij})^2 \right) \left| \dot{x}_j^T P_j x_j + x_j^T b_j^T x_j \right| \right]
\]

(21)

Define \( \Pi(\delta) = \Lambda_i I - \delta N P_i - \Lambda_i^T B_i^T B_i \). If \( \Pi(\delta) > 0 \) and (15a), (15b) hold for all \( i, l \), then \( \dot{V}(x(t)) < 0 \). Thus, the whole closed-loop fuzzy time-delay large-scale system is asymptotically stable if (15) holds, that is, \( \Pi(\delta) > 0 \), then by continuity, there is a \( \delta = 1 + \sigma \) with \( \sigma > 0 \) sufficiently small such that \( \Pi(\delta) > 0 \) for all \( i, l \). The proof is completed here. ■

**Remark 1.** By Schur complement, Theorem 1 can be easily transformed into the problem of LMIs. From Eq. (15), let \( \Omega^l_j > q_j, \Xi^l_j > q_j, \Theta^m_i > q_i, X_i = P_i^{-1} \), then inequalities (14) and (15a), (15b) are equivalent to the following LMIs

\[
U_j^T > X_i,
\]

\[
X_i A_i^T + A_i^T X_i + X_i q_j I X_i - \rho_i^l B_i^T B_i^T
\]

\[
- B_i^T B_i^T \rho_i^l + \left( I + \sum_{j=1}^{N} A_{ij}^T A_{ij}^T \right) < 0
\]

for all \( i = m \) (23a)

\[
X_i A_i^T + A_i^T X_i + X_i q_j I X_i + \left( I + \sum_{j=1}^{N} A_{ij}^T A_{ij}^T \right)
\]

\[
- \rho_i^l (B_i^T B_i^T + B_i B_i^T) - \frac{(B_i^T B_i^T + B_i B_i^T)}{2} \rho_i^l < 0
\]

for all \( i < m \) (23b)

**Remark 2.** The procedure to find the suitable \( P_i, \rho_i^l, U_i^l, K_i^l, q_j, \) and \( b_i \).

**Step 1.** Let \( q_i = 0 \).

**Step 2.** The researchers use LMI’s tool to solve the inequalities (23a), (23b). If the solution exists, that is, the common matrix \( X_i \) exists, then one can obtain the \( \rho_i^l, U_i^l, \)
and $P_i = X_i^{-1}$. Next, one can solve the $K_i^j = \rho_i^j B_i^T P_i$ and $b_i$ from the conditions (15c). Go to the next step. If the solution doesn’t exist, go to step 4.

**Step 3**: $q_i = q_i + 0.1$. Go to step 2.

**Step 4**: List all solutions under the different $q_i$. We can choose the most suitable $P_i$, $\rho_i^j$, $U_i^j$, $K_i^j$, and $b_i$.

If all the time-delays $\tau_i^j(t)$ are the same for all rules (i.e., $\tau_i^j(t) = \tau_i^j(t) = \tau_i^j$ for all $l \neq m$ and $\tau_i^j$ is constant for $i, j = 1, 2, \ldots, N$), then the equations (6) can be rewritten to the following:

$$
\dot{x}_i(t) = \sum_{l=1}^N h_i^l h_i^m \left( G_i^m x_i(t) + \sum_{j=1}^N \left( A_j^m x_j(t - \tau_i^j) + f_j^m \left( x_j(t), t \right) \right) \right)
$$

(24)

**Theorem 2**: Consider a fuzzy time-delay large-scale system $S$ composed of $N$ subsystems $S_i$ as (24). Suppose the assumption A1) holds and the state feedback gain is $K_i^j = \rho_i^j B_i^T P_i$, where $P_i > 0$ is the solution of (14). Then the overall closed-loop fuzzy time-delay system composed of $N$ subsystems $S_i$ (24) is stabilized asymptotically by the type-I fuzzy controller (4) if there exist $P_i$, $\rho_i^j$, and $Q_i^j$ satisfying the conditions, respectively.

$$
\hat{Q}_i^j = Q_i^j + (2\rho_i^j - 1) P_i B_i^T B_i^T P_i
$$

$$- P_i \left( I + \sum_{j=1}^N A_j^m A_j^m \right) P_i > 0 \text{ for all } l = m
$$

(25a)

$$
\hat{X}_i^j = X_i^j - P_i B_i^m B_i^T P_i - P_i \left( I + \sum_{j=1}^N A_j^m A_j^m \right) P_i
$$

$$+ \rho_i^j P_i (B_i^m B_i^T + B_i^m B_i^T) P_i > 0
$$

$$\hat{R}_i^m = R_i^m - P_i B_i^m B_i^T P_i - P_i \left( I + \sum_{j=1}^N A_j^m A_j^m \right) P_i
$$

$$+ \rho_i^j P_i (B_i^m B_i^T + B_i^m B_i^T) P_i > 0
$$

(25b)

$$N(1 + b_i^j) \leq \tilde{\Lambda}_i
$$

(25c)

in which $\tilde{\Gamma}_i^j = \min_{i} \lambda(\hat{Q}_i^j)$, $\tilde{\Gamma}_i^j = \min_{i} \lambda(\hat{X}_i^j)$, $\tilde{\Gamma}_i^j = \min_{i} \lambda(\hat{R}_i^m)$, and $\tilde{\Lambda}_i = \min_{i} (\tilde{\Gamma}_i^j, \tilde{\Gamma}_i^j, \tilde{\Gamma}_i^j)$, $b_i = \max_{1 \leq i, j \leq N} b_i^j$, for $i, j = 1, 2, \ldots, N, l = 1, 2, \ldots, r$, respectively.

**Proof**: Let the Lyapunov function for the closed-loop system composed of $N$ subsystem $S_i$ (24) be as follows:

$$
V = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \sum_{j=1}^{N} f_j^m \left( (0) x_i(0) d\theta \right) \right\}
$$

$$= \sum_{i=1}^{N} V_i(x_i), \quad i = 1, 2, \ldots, N.
$$

(26)

where $P_i > 0$ satisfies (14). Then the time derivative of $V(x_i)$ is

$$
\dot{V} = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \sum_{j=1}^{N} f_j^m \left( (0) x_i(0) d\theta \right) \right\}
$$

$$+ x_i^T P_i \left( I + \sum_{j=1}^N A_j^m A_j^m \right) P_i x_i
$$

$$+ \sum_{j=1}^{N} \left( h_j^m \left[ x_i^T Q_i^j x_i + x_i^T P_i B_i^T B_i^T P_i x_i \right. \right.

$$- \rho_i^j x_i^T P_i (B_i^m B_i^T + B_i^m B_i^T) P_i x_i
$$

(27)

Using Lemma 1, 2, the inequality (14), and assumption A1), one gets

$$
\dot{V} \leq \sum_{i=1}^{N} \left\{ \sum_{j=1}^{r} (h_j^m)^2 \left[ -x_i^T Q_i^j x_i + x_i^T P_i B_i^T B_i^T P_i x_i \right. \right.

$$+ x_i^T P_i \left( I + \sum_{j=1}^N A_j^m A_j^m \right) P_i x_i
$$

$$+ \sum_{j=1}^{N} \left( h_j^m \left[ x_i^T (t - \tau_i^j) I_{x_i}(t - \tau_i^j) \right. \right)

$$+ x_i^T (1 + b_i^j) I_{x_i}(t - \tau_i^j) I_{x_i}(t - \tau_i^j) \right. \left. \right)

$$+ \sum_{j=1}^{N} \left( h_j^m \left[ -x_i^T Q_i^j x_i + x_i^T P_i B_i^T B_i^T P_i x_i \right. \right.

$$- \rho_i^j x_i^T P_i (B_i^m B_i^T + B_i^m B_i^T) P_i x_i
$$

(28)
If Eqs. (25) hold, then 

\[
\begin{aligned}
V^\prime &< \sum_{j=1}^{N} \left\{ \left[ \sum_{i=1}^{N} (h_i^T)^2 (x_i^T Q_{ij} x_i) + \sum_{j=1}^{N} (h_i^T)^2 (1 + b_i^2) I_{x_i} \right] + \left[ \sum_{i=1}^{N} h_i^T \hat{A}_{ij} x_i + \sum_{j=1}^{N} h_i^T \hat{A}_{i} x_i \right] + \left( I + \sum_{j=1}^{N} A_{ij} A_{ij}^T \right) \right\} \n < 0
\end{aligned}
\]  

(29)

If \( \hat{\Omega}_i > 0, \hat{\Xi}_i > 0, \) and \( \hat{T}_i > 0, \) then we have 

\[
\begin{aligned}
V^\prime &< \sum_{j=1}^{N} \left\{ \left[ -x_i^T A_i x_i + x_i^T N(1 + b_i^2) I_{x_i} \right] \right\} 
\end{aligned}
\]  

(30)

If Eqs. (25) hold, then \( \dot{V}(x_i) < 0. \) The proof is completed here.

Remark 3. By Schur complement, Theorem 2 can be easily transformed into a problem of LMI’s. From Eq. (25), let \( \hat{\Omega}_i \) replace \( Q_{ij} \) in (15c). \( \hat{\Omega}_i \) is more conservative than \( Q_{ij} \) when \( \lambda \) is replaced by \( \hat{\lambda}_i \). In the following theorem, conditions are proposed to stabilize the system (1) using a type-II fuzzy controller.

Theorem 3. Consider a fuzzy time-delay large-scale system \( S \) as (1). Suppose the assumption A1) holds and the state feedback gain is \( K_i = \rho_i B_i^T P_i, \) where \( P_i > 0 \) is the solution of (32) as following:

\[
\begin{aligned}
\frac{A_i^T}{P_i} P_i + P_i \frac{A_i^T}{P_i} - P_i B_i^T B_i^T P_i + Q_i^T \leq 0
\end{aligned}
\]  

(32)

where \( Q_i^T = Q_i^T > 0. \) Then the overall closed-loop fuzzy time-delay system composed of \( N \) subsystems \( S_i \) is stabilized asymptotically by the type-II fuzzy controller (7) if there exist \( P_i, \rho_i, \hat{Q}_i, \) and \( U_i \) satisfying the following conditions, respectively.

\[
\begin{aligned}
P_i > U_i^{-1} \\
\hat{Q}_i = Q_i^T + (2 \rho_i - 1) P_i B_i^T B_i^T P_i - P_i \left( I + \sum_{j=1}^{N} A_{ij} A_{ij}^T \right) P_i > 0
\end{aligned}
\]  

(33a)

\[
\begin{aligned}
\hat{\xi}_i = Q_i^T + \rho_i P_i (B_i^T B_i^T + B_i^T B_i^T) P_i - P_i B_i^T B_i^T P_i > 0
\end{aligned}
\]  

(33b)

\[
\begin{aligned}
\hat{\lambda}_i = Q_i^T + \rho_i (B_i^T B_i^T + B_i^T B_i^T) - P_i B_i^T B_i^T P_i \geq 0
\end{aligned}
\]  

(33c)

in which \( b = \max_{i,j} b_{ij} \), for \( i,j = 1, 2, \ldots, N \), \( l = 1, 2, \ldots, r_n \), respectively.

Proof. The proof is similar to that of Theorem 1.

Remark 4. Consider a fuzzy time-delay large-scale system \( S \) as (1). Suppose the assumption A1) holds and the state feedback gain is \( K_i = \rho_i B_i^T P_i, \) where \( P_i > 0 \) is the solution of (32) as following:

\[
\begin{aligned}
\frac{A_i^T}{P_i} P_i + P_i \frac{A_i^T}{P_i} - P_i B_i^T B_i^T P_i + Q_i^T \leq 0
\end{aligned}
\]  

(32)

where \( Q_i^T = Q_i^T > 0. \) Then the overall closed-loop fuzzy time-delay system composed of \( N \) subsystems \( S_i \) is stabilized asymptotically by the type-II fuzzy controller (7) if there exist \( P_i, \rho_i, \hat{Q}_i, \) and \( U_i \) satisfying the following conditions, respectively.
If all the time-delays \( \tau_j(t) \) are the same for all rules (i.e., \( \tau_j(t) = \tau_j \) for all \( l \neq m \) and \( \tau_j \) is constant for \( l = 1, 2, \ldots, N \)), then the equations (8) can be rewritten as follows:

\[
\dot{x}_j(t) = \sum_{i=1}^{N} \sum_{n=1}^{r} h_i^n \{ \hat{G}_{ij}^n x_i(t) \} + \sum_{i=1}^{N} h_i^l \left[ \sum_{j=1}^{r} A_{ij}^l x_j(t - \tau_j) + f_j^l \left( x_j(t), t \right) \right]
\]

(35)

**Theorem 4.** Consider a fuzzy time-delay large-scale system \( S \) composed of \( N \) subsystem \( S_i \) as (35). Suppose the assumption A1) holds and the state feedback gain is \( K_i = \rho_i B_i^T P_i \), where \( P_i = P_i^T \) is a solution of (32). Then the overall closed-loop fuzzy time-delay system composed of \( N \) subsystem \( S_i \) (35) is stabilized asymptotically by the type-II fuzzy controller (7) if there exist \( P_i, \rho_i, \) and \( Q_i^l \) satisfying the following conditions, respectively.

\[
\dot{X}_i = Q_i^l + (2\rho_i - 1) P_i B_i^T J^T P_i - \rho_i I \sum_{j=1}^{r} A_i^j A_i^j T P_i > 0
\]

for all \( l = m \) (36a)

\[
\dot{X}_i = Q_i^l + \rho_i P_i (B_i^T J^T + B_i^T J^T) P_i - P_i B_i^T J^T P_i > 0
\]

(36b)

\[
N(1 + h^2) \leq \min \lambda(\hat{X}_i)
\]

(36c)

in which \( h_i = \max h^{b_i} \), for \( i, j = 1, 2, \ldots, N, l = 1, 2, \ldots, r, \) respectively.

**Proof.** The proof is similar to that of Theorem 2.

**Remark 6.** By Schur complement, Theorem 4 can be easily transformed into a problem of LMIs. From Eq. (36), let \( \hat{X}_i > q_i I, \dot{X}_i > q_i I, \hat{\Theta}_i > q_i I, \Theta_i > q_i I, X_i = P_i^{-1}, \) then the inequalities (32) and (36a), (36b) are equivalent to the following LMIs

\[
X_i A_i^T T + A_i^T T X_i + q_i I X_i - \rho_i B_i^T J^T T - B_i^T J^T P_i^T < 0
\]

(37a)

**IV. DELAY-DEPENDENT STABILIZATION OF FUZZY TIME-DELAY LARGE-SCALE SYSTEM**

In this section, the system (1) with \( \tau_j(t) = \tau_j^0(t) = \tau_j \) for all \( l \neq m \) and without nonlinear perturbation is considered and a delay-dependent stabilization criterion is proposed. According to the relation: \( x_j(t - \tau_j) = x_j(t) - \int_{t-\tau_j}^t \dot{x}_j(\theta) d\theta \), the closed-loop fuzzy system (24) without the nonlinear perturbation can be rewritten as

\[
\dot{x}_i(t) = \sum_{i=1}^{N} \sum_{n=1}^{r} h_i^n h_i^n P_i \left[ \frac{1}{N} G_{ij}^n x_i(t) + A_{ij}^T x_j(t) \right]
\]

\[
- \int_{t-\tau_j}^t A_{ij}^T \dot{x}_j(\theta) d\theta
\]

(38)

**Theorem 5.** The fuzzy time-delay system (38) can be stabilized asymptotically by a fuzzy control (4) with the feedback gain \( K_i = \rho_i B_i^T P_i \) (where \( P_i \) is the same as \( P_i \) of the equation (14)), if there exist some symmetric and positive definite matrices \( P_{ii}, P_{ij}, P_{ji}, P_{jj}, \rho_i^l > 0, \) \( R_{ij} = R_{ij}^T > 0, \) \( R_j = R_j^T, U_j^l > 0, \) and \( V_j > 0 \) such that the following conditions hold.

\[
\begin{bmatrix} R_{ii} & R_j \\ R_j & P_{jj} \end{bmatrix} \geq 0, \text{ for all } i, j
\]

(39a)

\[
\Sigma_j^l = -P_{ii} + U_j^l + (N + 1) \hat{\theta} A_{ij}^l P_{ii} A_{jj}^T < 0, \text{ for all } i, j, \text{ and } l
\]

(39b)

\[
A_{ij}^l = -Q_i^l - (2\rho_i^l - 1) P_{ii} B_i^T P_{ij} + N \tau_j R_{ii}
\]

(39c)
\[ V = V_1 + V_2 + V_3, \]
\[ V_1 = \sum_{i=1}^{N} \phi_i(t) P_{il} x_i(t), \quad V_2 = \sum_{i=1}^{N} \int_{t_{i-1}}^{t} \phi_i(t) P_{il} x_i(t) \, d\theta, \]
\[ V_3 = \sum_{i=1}^{N} \int_{t_{i-1}}^{t} \mathbf{x}_i^T(\theta) \mathbf{A}_i \mathbf{x}_i(\theta) \, d\theta. \]

Taking the derivative of \( V_1 \), one gets
\[ \dot{V}_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \]
\[ + 2 \phi_i(t) P_{il} A_{ij} x_j(t) - 2 \int_{t_{j-1}}^{t} \phi_i(t) P_{il} A_{ij} x_j(\theta) \, d\theta \]
\[
\begin{align*}
\dot{V}_1 & \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \\
& \quad + \phi_i(t) P_{il} A_{ij} x_j(t) \\
& \quad + \int_{t_{j-1}}^{t} x_j^T(\theta) \left[ \begin{array}{c}
R_{ii} \\
R_{ij} \\
R_{jj}
\end{array} \right] \left[ \begin{array}{c}
R_{ij} \\
R_{ij} \\
R_{jj}
\end{array} \right] x_i(\theta) \, d\theta
\end{align*}
\]

By using Lemma 4 and letting \( v_1(0) = x_i(t), v_2(0) = \dot{x}_i(0) \), one gets
\[ V_1 \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \]
\[ + 2 \phi_i(t) P_{il} A_{ij} x_j(t) \]
\[ + \int_{t_{j-1}}^{t} x_j^T(\theta) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_i(\theta) \, d\theta \]
\[
\begin{align*}
\dot{V}_1 & \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \\
& \quad + 2 \phi_i(t) P_{il} A_{ij} x_j(t) + \int_{t_{j-1}}^{t} x_j^T(\theta) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_i(\theta) \, d\theta
\end{align*}
\]

where \[ \begin{bmatrix} R_{ii} & R_{ij} \\
R_{ij} & R_{jj} \end{bmatrix} \geq 0. \]

Taking the derivative of \( V_2(x(t)) \), one gets
\[ V_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ \phi_i^T(t) P_{il} x_i(t) - \phi_j^T(t) (P_{il} A_{ij} - R_{ij}) x_j(t) \right] \]
\[ = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ \phi_i^T(t) P_{il} x_i(t) - \phi_j^T(t) (P_{il} A_{ij} - R_{ij}) x_j(t) \right] \]
\[
\begin{align*}
\dot{V}_3 & \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \\
& \quad + 2 \phi_i(t) P_{il} A_{ij} x_j(t) + \phi_i(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_j(t) \\
& \quad + \int_{t_{j-1}}^{t} x_j^T(\theta) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_i(\theta) \, d\theta
\end{align*}
\]

Taking the derivative of \( V_3 \), one gets
\[ V_3 = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ \phi_i^T(t) P_{ij} \dot{x}_j(t) - \phi_j^T(t) (P_{ij} A_{ij} - R_{ij}) \dot{x}_j(t) \right] \]
\[
\begin{align*}
\dot{V}_3 & \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \\
& \quad + 2 \phi_i(t) P_{il} A_{ij} x_j(t) \]
\[
\begin{align*}
V & = V_1 + V_2 + V_3 \\
& \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{m} h_l h_m \left[ x_i^T(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) \right] x_j(t) \\
& \quad + 2 \phi_i(t) P_{il} A_{ij} x_j(t) + \phi_i(t) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_j(t) \\
& \quad + \int_{t_{j-1}}^{t} x_j^T(\theta) \left( \frac{1}{N} \left( G_{il}^T P_{il} + P_{il} G_{il}^T \right) \right) x_i(\theta) \, d\theta
\end{align*}
\]
Substituting $K_j^i = \rho |B|^2 P_{ij}$ and (14) into (51) and according to (39b), (39c), (39d), (39e), one gets

$$
\dot{V} \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} (h_{ij}^l)^2 \left\{ \frac{1}{N} \vec{x}_i^T(t) \left[ G_{ij}^{mT} P_{ii} + P_{ij} G_{ij}^{m} + N \tau_{ij} R_{ii} + N P_2 \right] + (N+1) \bar{\varepsilon}_i G_{ij}^{mT} P_{ij} G_{ij}^{m} + 2NR_2 \right. \\
+ N(P_{ii} A_{ij}^l - R_2) U_i^{-1} (P_{ij} A_{ij}^l - R_2)^T \right\} x_i(t) \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} (h_{ij}^l)^2 \left\{ x_i^T(t) \left[ \frac{1}{N} \Lambda_{ij}^{mT} P_{ij} + P_{ij} G_{ij}^{mT} + N \tau_{ij} R_{ij} \right] + N \tau_{ij} R_{ij} + (N+1) \bar{\varepsilon}_i G_{ij}^{mT} P_{ij} G_{ij}^{m} + 2NR_2 \right. \\
+ N(P_{ij} A_{ij}^l - R_2) U_i^{-1} (P_{ij} A_{ij}^l - R_2)^T \right\} x_i(t) \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} (h_{ij}^l)^2 \left\{ (N+1) \bar{\varepsilon}_i A_{ij}^l P_{ij} A_{ij}^l - P_{ij} + U_i^T \right\} x_i(t) \\
\left( \varepsilon_i^T \right) \\
$$

Substituting $K_j^i = \rho |B|^2 P_{ij}$ and (14) into (51) and according to (39b), (39c), (39d), (39e), one gets

$$
\dot{V} \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} (h_{ij}^l)^2 \left\{ \frac{1}{N} \vec{x}_i^T(t) \left[ G_{ij}^{mT} P_{ii} + P_{ij} G_{ij}^{m} + N \tau_{ij} R_{ii} \right] + (N+1) \bar{\varepsilon}_i G_{ij}^{mT} P_{ij} G_{ij}^{m} + 2NR_2 \right. \\
+ N(P_{ii} A_{ij}^l - R_2) U_i^{-1} (P_{ij} A_{ij}^l - R_2)^T \right\} x_i(t) \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} (h_{ij}^l)^2 \left\{ \tau_{ij} x_i(t) \left[ \frac{1}{N} \Lambda_{ij}^{mT} P_{ij} + P_{ij} G_{ij}^{mT} + N \tau_{ij} R_{ij} \right] + N \tau_{ij} R_{ij} + (N+1) \bar{\varepsilon}_i G_{ij}^{mT} P_{ij} G_{ij}^{m} + 2NR_2 \right. \\
+ N(P_{ij} A_{ij}^l - R_2) U_i^{-1} (P_{ij} A_{ij}^l - R_2)^T \right\} x_i(t) \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} (h_{ij}^l)^2 \left\{ (N+1) \bar{\varepsilon}_i A_{ij}^l P_{ij} A_{ij}^l - P_{ij} + U_i^T \right\} x_i(t) \\
\left( \varepsilon_i^T \right) \\
$$

Remark 8. By Schur complement, Theorem 5 can be easily transformed into a problem of LMIs. Pre- and post-multiply $\{ P_{ij} \}$, $\{ P_{ij}^{-1} \}$ to (39a), (39b), (39c), (39d), (39e), let $X_{ii} = P_{ii}^{-1}$, $X_{ji} = P_{ji}^{-1}$, $M_i = X_i X_i$, $H_i = X_i L_i X_i$, and then inequalities (14) and (39) are equivalent to the following LMIs

$$
\begin{bmatrix}
L_{il} & L_{2i} \\
L_{2i} & 2X_{ii} - X_{jj}
\end{bmatrix} \geq 0
$$

$$
\begin{bmatrix}
M_i - H_i \\
\sqrt{(N+1) \bar{\varepsilon}_i} A_{ij}^T X_{ii} - X_{ii}
\end{bmatrix} < 0
$$

$$
\begin{bmatrix}
Y_{ij} \cdot Y_{ij}^T + 2X_{ii} - X_{jj} \\
\sqrt{(N+1) \bar{\varepsilon}_i} A_{ij}^T X_{ii} - X_{ii}
\end{bmatrix} < 0
$$

$$
\begin{bmatrix}
G_i^T - H_i \\
\sqrt{(N+1) \bar{\varepsilon}_i} A_{ij}^T X_{ii} - X_{ii}
\end{bmatrix} < 0
$$

$$
\begin{bmatrix}
Y_{ij} \cdot Y_{ij}^T + 2X_{ii} - X_{jj} \\
\sqrt{(N+1) \bar{\varepsilon}_i} A_{ij}^T X_{ii} - X_{ii}
\end{bmatrix} < 0
$$

If $\Lambda_{ij}^{m} < 0$, $\bar{\Lambda}_{ij}^{m} < 0$, $\sum_{i} < 0$ and $\bar{\sum}_{i} < 0$ holds for all $i$, $j$, $l$, and $m$, then $\dot{V} < 0$. Thus, the whole fuzzy time-delay closed loop large scale system is asymptotically stable. ■
By using LMIs, we can obtain the suitable control gain $K_i = \rho_i B_i^T P_i$ and (53a), (53b), (53c), (53d), (53e).

In the following theorem, the criteria are proposed to stabilize the closed-loop fuzzy system (35) without the nonlinear perturbations using a type-II controller.

**Theorem 6.** The fuzzy time-delay system (35) without nonlinear perturbations can be stabilized asymptotically using fuzzy control (7) with the feedback gain $K_i = \rho_i B_i^T P_i$ (where $P_i$ is the same as $P_i$ of the equation (32), if there exist some symmetric and positive definite matrices $P_1, P_2, P_3, \rho_i > 0, R_i = R_i^T > 0, R_2 = R_2^T$, and $U_i > 0$ such that the following conditions hold:

(a) $\begin{bmatrix} R_i & R_2 \\ R_2 & P_3 \end{bmatrix} \geq 0,$ for all $i$ and $j,$ (54a)

(b) $\Sigma_{ij} = -P_2 + U_i^T + (N + \tau_i) \tilde{A}_i^T P_3 A_j^T < 0,$ for all $i, j,$ and $l,$ (54b)

(c) $A_{ij} = -Q_{ij} - (2\rho_i - 1) P_i B_i^T B_i^T P_i + N\tau_i R_i + 2 N R_2 + N P_2 + (N + \tau_i) \tilde{\tau}_i G_i^T P_i G_i^T$

$+ N (P_i A_j^T - R_2) U_i^{-1} (P_i A_j^T - R_2)^T < 0,$ for all $i, j,$ and $l,$ (54c)

Proof. The proof is similar to that of Theorem 5.

**Remark 9.** By Schur complement, Theorem 6 can be easily transformed into a problem of LMIs. Pre- and post-multiply $\{ P_1^{-1}, P_1^{-1} \}$ to (54a), (54b), (54c), (54d), let $X_i = P_i^{-1}, X_i = P_i^{-1}, L_i = X_i R_i X_i, L_2 = X_i R_i X_i, M_i = X_i U_i^T X_i, M_i = X_i U_i^T X_i, H_i = X_i P_2 X_i,$ and $X_i X_i^T X_i > 2 X_i - X_i$ (by Lemma 5), then inequalities (32) and (54) are equivalent to the following LMIs

\[
\begin{bmatrix}
L_i & -L_i \\
L_2 & 2 X_i - X_2
\end{bmatrix} \geq 0
\]

\[
\begin{bmatrix}
M_i - H_i & \sqrt{(N + \tau_i) \tilde{\tau}_i} A_i^T X_i \\
\sqrt{(N + \tau_i) \tilde{\tau}_i} A_i^T X_i & -X_i
\end{bmatrix} < 0
\]

(55b)
By using LMI’s tool, one can obtain the suitable control gains $K_i = P_i^T B_i P_i$ and (55a), (55b), (55c), (55d).

**Remark 10.** In general, delay-independent stabilization criteria are more conservative than that of delay-dependent, especially when the size of time delay is actually small.

### V. AN ILLUSTRATIVE EXAMPLE AND SIMULATION

**Example.** Consider a large-scale system $S$ composed of three fuzzy subsystems $S_i$ as:

**Subsystem 1.**

- Rule 1: If $x_{11}(t)$ is about 0 and $x_{12}(t)$ is about 0,
  
  \[
  \dot{x}_1(t) = \begin{bmatrix} -3 & 3 \\ 2 & -5 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix} u_1 
  + \sum_{j=1}^3 A_{1j}^1 x_j(t - \tau_{1j}(t)) + f_{1j}^1(x_j(t), t). 
  \]

- Rule 2: If $x_{11}(t)$ is about 0 and $x_{12}(t)$ is about ±1,

  \[
  \dot{x}_1(t) = \begin{bmatrix} -4 & 2 \\ 3 & -3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix} u_1 
  + \sum_{j=1}^3 A_{1j}^2 x_j(t - \tau_{1j}(t)) + f_{1j}^2(x_j(t), t). 
  \]

- Rule 3: If $x_{11}(t)$ is about ±1 and $x_{12}(t)$ is about 0,

  \[
  \dot{x}_1(t) = \begin{bmatrix} -5 & 3 \\ 2 & -2 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.85 \\ 0.85 \end{bmatrix} u_1 
  + \sum_{j=1}^3 A_{1j}^3 x_j(t - \tau_{1j}(t)) + f_{1j}^3(x_j(t), t). 
  \]

**Subsystem 2.**

- Rule 1: If $x_{21}(t)$ is about 0 and $x_{22}(t)$ is about 0,

  \[
  \dot{x}_2(t) = \begin{bmatrix} -2.25 & 6 \\ 1.5 & -3.75 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix} u_2 
  + \sum_{j=2}^3 A_{2j}^1 x_j(t - \tau_{2j}(t)) + f_{2j}^1(x_j(t), t). 
  \]

- Rule 2: If $x_{21}(t)$ is about ±1 and $x_{22}(t)$ is about −1,

  \[
  \dot{x}_2(t) = \begin{bmatrix} -3.75 & 4.5 \\ 1.5 & -1.5 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix} u_2 
  + \sum_{j=2}^3 A_{2j}^2 x_j(t - \tau_{2j}(t)) + f_{2j}^2(x_j(t), t). 
  \]

**Subsystem 3.**

- Rule 1: If $x_{31}(t)$ is about 0 and $x_{32}(t)$ is about 0,

  \[
  \dot{x}_3(t) = \begin{bmatrix} -4.8 & 6 \\ 3.6 & -3.6 \end{bmatrix} x_3(t) + \begin{bmatrix} 0.96 \\ 0.96 \end{bmatrix} u_3 
  + \sum_{j=3}^3 A_{3j}^1 x_j(t - \tau_{3j}(t)) + f_{3j}^1(x_j(t), t). 
  \]

- Rule 2: If $x_{31}(t)$ is about ±2 and $x_{32}(t)$ is about −1,

  \[
  \dot{x}_3(t) = \begin{bmatrix} -6 & 7.2 \\ 2.4 & -2.4 \end{bmatrix} x_3(t) + \begin{bmatrix} 1.02 \\ 1.02 \end{bmatrix} u_3 
  + \sum_{j=3}^3 A_{3j}^2 x_j(t - \tau_{3j}(t)) + f_{3j}^2(x_j(t), t). 
  \]

The membership functions for $x_{11}$, $x_{12}$, $x_{21}$, $x_{22}$, $x_{31}$, and $x_{32}$ are shown in Fig. 1 ~ 3, respectively. The interconnections among three subsystems are given as:

\[
\begin{align*}
A_{12}^1 &= \begin{bmatrix} 0.5 & 0.28 \\ 0.4 & 0.18 \end{bmatrix}, & A_{13}^1 &= \begin{bmatrix} 0.11 & 0.32 \\ 0.21 & 0.43 \end{bmatrix}, \\
A_{12}^2 &= \begin{bmatrix} 0.22 & 0.5 \\ 0.25 & 0.55 \end{bmatrix}, & A_{13}^2 &= \begin{bmatrix} 0.3 & 0.3 \\ 0.28 & 0.28 \end{bmatrix}, \\
A_{12}^3 &= \begin{bmatrix} 0.3 & 0.4 \\ 0.3 & 0.3 \end{bmatrix}, & A_{13}^3 &= \begin{bmatrix} 0.18 & 0.27 \\ 0.2 & 0.18 \end{bmatrix}, \\
A_{21}^1 &= \begin{bmatrix} 0.24 & 0.3 \\ 0.2 & 0.28 \end{bmatrix}, & A_{23}^1 &= \begin{bmatrix} 0.3 & 0.25 \\ 0.28 & 0.24 \end{bmatrix}, \\
A_{21}^2 &= \begin{bmatrix} 0.18 & 0.36 \\ 0.23 & 0.32 \end{bmatrix}, & A_{23}^2 &= \begin{bmatrix} 0.3 & 0.3 \\ 0.32 & 0.26 \end{bmatrix}, \\
A_{21}^3 &= \begin{bmatrix} 0.22 & 0.15 \\ 0.25 & 0.18 \end{bmatrix}, & A_{23}^3 &= \begin{bmatrix} 0.3 & 0.15 \\ 0.3 & 0.15 \end{bmatrix}, \\
A_{31}^1 &= \begin{bmatrix} 0.2 & 0.4 \\ 0.18 & 0.3 \end{bmatrix}, & A_{32}^1 &= \begin{bmatrix} 0.27 & 0.135 \\ 0.25 & 0.14 \end{bmatrix}.
\end{align*}
\]

Fig. 1. The membership function of $x_{11}$ and $x_{12}$.
Moreover, \( f_i^j(x_j(t), t) = \begin{bmatrix} b_{ij}^1 x_{ij}(t) \sin \left( x_{ij}(t) + \frac{\pi}{2} \right) \\ b_{ij}^2 x_{ij}(t) \sin \left( x_{ij}(t) - \frac{\pi}{2} \right) \end{bmatrix} \) satisfies (A2) for all \( i, j \). Using the approach of Theorem 1, Remark 1, 2, and LMI tool, one gets \( q_1 = 0.1, q_2 = 0.6, q_3 = 0.8 \) and feedback gains as following:

\[
\begin{align*}
K_1^1 &= \begin{bmatrix} 24.2788 & 22.3973 \end{bmatrix}, \\
K_1^2 &= \begin{bmatrix} 24.2788 & 22.3973 \end{bmatrix}, \\
K_1^3 &= \begin{bmatrix} 24.2788 & 22.3973 \end{bmatrix}, \\
K_2^1 &= \begin{bmatrix} 9.1276 & 9.6570 \end{bmatrix}, \\
K_2^2 &= \begin{bmatrix} 9.1276 & 9.6570 \end{bmatrix}, \\
K_2^3 &= \begin{bmatrix} 3.6328 & 4.0573 \end{bmatrix}, \\
K_3^1 &= \begin{bmatrix} 3.6328 & 4.0573 \end{bmatrix}.
\end{align*}
\]

Checking the inequality (15c), the tolerable bounds are \( b_1 = 0.2483, b_2 = 0.9534, \) and \( b_3 = 0.3710 \). Using the approach of Theorem 2, Remark 2, 3 and LMI tool, one gets \( q_1 = 0.1, q_2 = 0.6, q_3 = 0.8, \) and feedback gains as following:

\[
\begin{align*}
K_1^1 &= \begin{bmatrix} 7.0545 & 6.2326 \end{bmatrix}, \\
K_1^2 &= \begin{bmatrix} 7.0545 & 6.2326 \end{bmatrix}, \\
K_1^3 &= \begin{bmatrix} 7.0545 & 6.2326 \end{bmatrix}, \\
K_2^1 &= \begin{bmatrix} 9.1276 & 9.6570 \end{bmatrix}, \\
K_2^2 &= \begin{bmatrix} 9.1276 & 9.6570 \end{bmatrix}, \\
K_2^3 &= \begin{bmatrix} 3.6328 & 4.0573 \end{bmatrix}, \\
K_3^1 &= \begin{bmatrix} 3.6328 & 4.0573 \end{bmatrix}.
\end{align*}
\]

Checking the inequality (25c), the tolerable bounds are \( b_1 = 2.9379, b_2 = 2.2099, \) and \( b_3 = 1.1713 \). Therefore, the design method of Theorem 1 is definitely more conservative than that of Theorem 2 in this example. Figure 4 shows the simulation results of three subsystems with time-delay \( \tau_{ij} = 1 \) for all \( i, j \).

\[
\begin{align*}
K_1^1 &= \begin{bmatrix} 1.6959 & 2.6928 \end{bmatrix}, \\
K_1^2 &= \begin{bmatrix} 1.7277 & 2.7433 \end{bmatrix}, \\
K_1^3 &= \begin{bmatrix} 1.7001 & 2.6995 \end{bmatrix}, \\
K_2^1 &= \begin{bmatrix} 2.6779 & 4.8704 \end{bmatrix}, \\
K_2^2 &= \begin{bmatrix} 2.6795 & 4.8734 \end{bmatrix}, \\
K_2^3 &= \begin{bmatrix} 1.5934 & 2.6926 \end{bmatrix}, \\
K_3^1 &= \begin{bmatrix} 1.5934 & 2.6926 \end{bmatrix}.
\end{align*}
\]

Figure 5 shows the simulation results of three subsystems with time-delay \( \tau_{ij} = 0.048 \) for all \( i, j \) and \( \psi_i(t) = x_i(0) \) for all \( i \).

\[
\begin{align*}
x_1^1(0) &= \begin{bmatrix} -0.1 \ 0.1 \end{bmatrix}^T, \\
x_2^1(0) &= \begin{bmatrix} 2 \ -0.6 \end{bmatrix}, \\
x_3^1(0) &= \begin{bmatrix} 1.5 \ -1 \end{bmatrix}^T
\end{align*}
\]

\[
\begin{align*}
x_1^2(0) &= \begin{bmatrix} -0.1 \ 0.1 \end{bmatrix}^T, \\
x_2^2(0) &= \begin{bmatrix} 2 \ -0.6 \end{bmatrix}, \\
x_3^2(0) &= \begin{bmatrix} 1.5 \ -1 \end{bmatrix}^T
\end{align*}
\]

\[
\begin{align*}
x_1^3(0) &= \begin{bmatrix} -0.1 \ 0.1 \end{bmatrix}^T, \\
x_2^3(0) &= \begin{bmatrix} 2 \ -0.6 \end{bmatrix}, \\
x_3^3(0) &= \begin{bmatrix} 1.5 \ -1 \end{bmatrix}^T
\end{align*}
\]

Fig. 2. The membership function of \( x_{21} \) and \( x_{22} \).

Fig. 3. The membership function of \( x_{31} \) and \( x_{32} \).
VI. CONCLUSIONS

Design approaches for guaranteeing stabilization of perturbed fuzzy time-delay large-scale systems have been proposed using two different decentralized state feedback controllers. The suitable control gains $K_i$ and perturbation bounds $b_i$ can be obtained easily using the procedure in Remark 2 and LMIs tool. Furthermore, the design algorithm shows that we can obtain the larger perturbation bounds $b_i$ by choosing a suitable $q_i$. Moreover, systems with all the time-delays $\tau_{ji}(t)$ are the same for all rules (i.e., $\tau_{ji}(t) = \tau_{ik}(t) = \tau_{jm}$ for all $l \neq m$), allow the authors to propose simpler and less conservative criteria. The delay-dependent criterion is also derived for the time-delay fuzzy large-scale systems without perturbations. The design methodology is illustrated by this application to a numerical example.

REFERENCES


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