ADAPTIVE COORDINATED DECENTRALIZED CONTROL OF STATE DELAYED SYSTEMS WITH ACTUATOR FAILURES

Boris M. Mirkin and Per-Olof Gutman

ABSTRACT

This paper develops an adaptive state feedback coordinated decentralized control scheme for a class of dynamic systems with state delay in subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem. The main contributions of this paper are the development of a new controller parametrization which attempt to anticipate the future states and failures, the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms, and a stability proof.

KeyWords: Adaptive decentralized control, interconnected time-delay systems, actuator failures.

I. INTRODUCTION

Control design for systems with actuator failures is an area of research that has been studied a lot in recent years. Many important results have been obtained. The designs have been based mainly on the following approaches: actuator fault detection and isolation methods [1-3], the robust fault accommodation approach [4], multiple model switching and tuning methods [5,6], and the sliding mode method [7]. One of the main research directions is adaptive control, see e.g. [8-12], and references therein.

In the recent series of papers, see, e.g. [10] and the books [11,12] the model reference adaptive control (MRAC) technique was successful applied for numerous problems with actuator failures in the centralized framework and in the delay-free case.

Yet, relatively few results using adaptive control for the important class of delayed systems with actuator failures are available in the literature. Time-delay is a natural component of dynamic processes in many engineering fields and its presence in the plants considerably complicates the design problem, see e.g. the recent papers [13,14] for centralized control cases. In [13] a fault detection and accommodation procedure is considered for stable nonlinear state delay plants, based on an iterative design of an observer which monitors the variations of the system dynamics. Within the framework of Linear Matrix Inequalities techniques, a robust state feedback linear controller $u = Kx(t)$ is designed for the stabilization of the linear plant with input delay, and actuator failures of stuck-type [14]. In [15] we proposed two adaptive state feedback control schemes for a class of linear systems with state delay in the presence of unknown actuator failures for the centralized control problem. To the best of the authors’ knowledge, the decentralized MRAC design problem for plants with actuator failures and time delay has not yet been solved.

In this paper, using a Lyapunov approach and our adaptive decentralized control scheme with model coordination, see, e.g. [16,17], we present a decentralized model reference adaptive controller (DMRAC) for a class of uncertain dynamic systems with state delays in the subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem. A special Lyapunov-Krasovskii functional is used to design the update mechanism for the controller parameters. For the updating of the controller parameters, we use a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes [15].
The main contributions of the paper are: (i) the enlargement of the class of systems with actuator failures that can be handled using model reference adaptive control; (ii) a direct coordinated decentralized adaptive control law parameterization which attempt to anticipate the future states and actuator failures in the subsystems; (iii) the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms and to prove stability.

The paper is organized as follows: In Section 2 the problem statement and preliminaries are presented. In Section 3 the decentralized controller parameterization with reference model coordination is presented. The basic tracking error equation is established in Section 4. The Section 5 includes the adaptive control scheme and the proof of stability. Some simulation results are presented in Section 6.

II. PLANT MODEL AND PROBLEM FORMULATION

We consider a class of uncertain systems, which are composed of $M$ multi-input multi-output subsystems with state delays in subsystems and in interconnections whose control components may fail at the time of operation described by equations, suitably initialized, of the form

$$\dot{x}_i(t) = A_i x_i(t) + A_{ii} x_i(t - \tau_i) + B_i u_{pi}(t) + \sum_{j=1, j \neq i}^M B_j A_j x_j(t - \tau_j)$$

$$u_{pi}(t) = \sum_{q=1}^{m_i} b_{iq} u_{iq}(t)$$

where, for the $i$-th subsystem $x_i \in \mathbb{R}^n$ is the state vector, $u_{pi}(t) = [u_{pi1}(t), \ldots, u_{pim_i}(t)]^T$ is the applied to the plant real control input vector, $b_{iq}, (q = 1, 2, \ldots, m_i)$ is the $q$th column of $B_i$, $\tau_i = [\tau_{i1}, \ldots, \tau_{i2}, \ldots, \tau_{im_i}]^T$ is some constant vector and $u_{pi}(t) = [u_{i1}(t), \ldots, u_{iq}(t), \ldots, u_{im_i}(t)]^T$ is the control vector to be designed. The constant matrices $A_i, B_i \in \mathbb{R}^{n \times n}, A_{ii} \in \mathbb{R}^{n \times n}, B_j \in \mathbb{R}^{n \times n}, A_j \in \mathbb{R}^{n \times n}$ have unknown elements. $\tau_j \in \mathbb{R}^+, i, l = 1, \ldots, M$ are known time delays and $\sum_{i=1}^M n_i = n$.

**Remark 1.** The factorization through $B_i$ in the plant interconnections is restrictive for general applications. However, e.g. mechanical systems with rate sensors whose inputs are forces and torques satisfy such conditions see, e.g [18]. The damping of elastic motions in flexible structures with co-located sensors and actuators makes for a control problem that satisfies the assumption see, e.g. [19].

The indicator matrix $Y_i = \text{diag}(\upsilon_{i1}, \ldots, \upsilon_{iqr}, \ldots, \upsilon_{iqm_i}) \in \mathbb{R}^{m_i \times m_i}$ describes the working condition of the actuators. $\upsilon_{iq} = 1$ denotes that the $q$th actuator of the $i$th system is in normal mode, and the actually applied $q$th control input to the subsystem is $u_{iq}$. $\upsilon_{iq} = 0$ denotes a failed actuator, e.g. stuck at certain position, and the actually applied $q$th control input to the subsystem equals $\bar{u}_{iq}$. If all actuators of the subsystem are in normal mode, the matrix $Y_i$ is an identity matrix $Y_i = I$. The constant value $\bar{u}_{iq}$ and the failure time instant $t_q$ are unknown, i.e. the type of actuator failures considered here is the same as in [9]:

$$u_{iq}(t) = \bar{u}_{iq}, \quad t \geq t_q, \quad q = 1, 2, \ldots, m_i$$ (2)

Note that it is postulated that a failed actuator never returns to normal operation. For the decentralized control problem considered in this paper, the first assumption is that (A1) in the case of known plant parameters and known actuator failures, there exists a decentralized controller such that the system (1) can be state feedback stabilized by the remaining actuators.

The problem is to design an adaptive feedback control, and tune, on-line, the controller parameters in order to achieve desired closed loop specifications when there are up to $m_i - 1$ unknown actuator failures. The desired specification in this paper is that with a failure model (2), all signals of the closed loop system remain bounded, and that the each subsystems state $x_i(t)$ asymptotically exact follows the state $x_i(t)$ of a stable reference model without delays

$$\dot{x}_i(t) = A_{ii} x_i(t) + b_{ii} r_i(t)$$ (3)

where, $A_{ii} \in \mathbb{R}^{n \times n}, b_{ii} \in \mathbb{R}^n$ are known constant matrices, and $r_i(t) \in \mathbb{R}$ is a bounded reference input signal. This means that we demand that $\lim_{t \to \infty} \| e(t) \| = \| x_i(t) - x_i(t) \|$ = 0, $i = 1, \ldots, M$, i.e. also in the presence of up to $m_i - 1$ actuator failures in each subsystem $i$. As in [10] for the centralized case, we assume that (A2) if the plant parameters and the actuator failures (up to $m_i - 1$ failures in each subsystem $i$) are known, the remaining subsystem actuators can still achieve the desired control objective.

III. CONTROLLER PARAMETERIZATION

Motivated by our previous works, see, e.g. [16,17,20], we will use the decentralized adaptive control scheme with reference model coordination to achieve the control objective. The control law for the $i$th local subsystem $u_i(t)$ is chosen to be of the form

$$u_i(t) = u_{pi}(t) + u_{ci}(t)$$ (4)

where the part of the control law $u_{pi}(t)$ is based only on the local signals of the $i$th subsystem, and the component $u_{ci}(t)$ is the coordinated component which is based on the refer-
ence signals of all the other subsystems. Exchange of the reference signals between subsystems can be easily implemented in real-life control systems.

The part of the control law $u_y(t)$ which is based only on the local information is parameterized as follows

$$u_y(t) = \Theta_{x_i}^T(t) e_i(t) + \Theta_{x_i}^T(t) x_i(t) + \Theta_{x_i}^T(t) x_i(t-\tau_i) + \Theta_{x_i}^T(t) r_i(t)$$

(5)

where $\Theta_{x_i}(t) = [\theta_{x_{i1}}(t), \ldots, \theta_{x_{i}}(t), \ldots, \theta_{x_{im}}(t)]^T \in \mathbb{R}^{n_{x_i} \times n_i}$, $\Theta_{ui}(t) = [\theta_{ui1}(t), \ldots, \theta_{ui}(t), \ldots, \theta_{uin}(t)]^T \in \mathbb{R}^{n_{ui} \times n_i}$, and $\Theta_{d}(t) = [\theta_{d1}(t), \ldots, \theta_{d}(t), \ldots, \theta_{dmin}(t)]^T \in \mathbb{R}^{n_{d} \times n_i}$.

The coordinated control component, $u_c(t)$, which is based on the reference signals of all the other subsystems is defined as

$$u_c(t) = \sum_{i \neq j \neq \ldots \neq k \neq l} \Theta_{x_{ij}}^T(t) e_j(t) + \Theta_{x_{ij}}^T(t) x_j(t) + \Theta_{x_{ij}}^T(t) x_j(t-\tau_j) + \Theta_{x_{ij}}^T(t) r_j(t)$$

where $\Theta_{x_{ij}}(t) = [\theta_{x_{i1}}(t), \ldots, \theta_{x_{i}}(t), \ldots, \theta_{x_{ij}}(t)]^T \in \mathbb{R}^{n_{x_{ij}} \times n_i}$, and the $q$th component of $u_c(t)$ is

$$u_{c_q}(t) = \sum_{i \neq j \neq \ldots \neq k \neq l} \Theta_{x_{ijq}}^T(t) e_j(t) + \Theta_{x_{ijq}}^T(t) x_j(t)$$

(7)

where

$$\Theta_{x_{ijq}}(t) = [\theta_{x_{i1}}(t), \ldots, \theta_{x_{i}}(t), \ldots, \theta_{x_{ijq}}(t)]^T \in \mathbb{R}^{n_{x_{ijq}} \times n_i}$$

and the $q$th component of $u_{c_q}(t)$, $q = 1, 2, \ldots, m_i$, is

$$u_{c_q}(t) = \sum_{i \neq j \neq \ldots \neq k \neq l} \Theta_{x_{ijq}}^T(t) x_j(t-\tau_j)$$

(8)

IV. PROPOSED ERROR EQUATION PARAMETRIZATION

4.1. Basic perfect model-following conditions for isolated subsystems

Based on [12,15] we briefly present model-following conditions for the plant without interconnections which are useful for error parametrization.

Let us assume that all the parameters of (1) and the actuator failures are known, and let us define $u_d(t)$ from (5) as

$$u_d^*(t) = \Theta_{x_i}^T(t) e_i(t) + \Theta_{x_i}^T(t) x_i(t) + \Theta_{x_i}^T(t) x_i(t-\tau_i) + \Theta_{x_i}^T(t) r_i(t)$$

(9)

where the constant matrices $\Theta_{x_i}^* = [\theta_{x_{i1}}^*, \theta_{x_{i2}}^*, \ldots, \theta_{x_{im}}^*] \in \mathbb{R}^{n_{x_i} \times n_i}$, $\Theta_{ui}^* = [\theta_{ui1}^*, \theta_{ui2}^*, \ldots, \theta_{uin}^*] \in \mathbb{R}^{n_{ui} \times n_i}$, $\Theta_{d}^* = [\theta_{d1}^*, \theta_{d2}^*, \ldots, \theta_{dmin}^*] \in \mathbb{R}^{n_{d} \times n_i}$ are to be defined for perfect model-following.

The $q$th component of $u_d$, $q = 1, 2, \ldots, m_i$, can be written

$$u_{d_q}^*(t) = \Theta_{x_{iq}}^T(t) e_i(t) + \Theta_{x_{iq}}^T(t) x_i(t) + \Theta_{x_{iq}}^T(t) x_i(t-\tau_i) + \Theta_{x_{iq}}^T(t) r_i(t)$$

(10)

Suppose there are $g_i$ failed actuators in the each sub-system, i.e.

$$u_{d_q}^*(t) = \overline{u}_{d_q}, \quad \text{for } j_1, j_2, \ldots, j_{g_i}, 1 \leq g_i \leq m_i - 1.$$

To meet the control objective, we assume that for each isolated subsystem there exist constant vectors $\theta_{x_{iq}}^*$, $\theta_{uiq}^*$, and nonzero constant scalars $\theta_{d}^*$, $\theta_{d}^*$ such that the following equations are satisfied

$$A_i - A_{ij} + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{iq} \Theta_{x_{ijq}}^* = 0,$$

$$A_{ij} + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{iq} \Theta_{x_{ijq}}^* - b_{ij} = 0,$$

$$\sum_{q \neq j \neq \ldots \neq k \neq l} b_{iq} \theta_{uiq}^* + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{iq} \overline{u}_{d_q} = 0$$

(11)

where $b_{iq} \in \mathbb{R}^{n_{uiq}}$ is the $q$th column of $B_i = [b_{i1}, \ldots, b_{iq}, \ldots, b_{iwm_i}]$ in (1).

Then using (10) and (11) we can write the closed-loop “plant-controller” system as

$$x_i(t) = A_i x_i(t) + A_{ij} x_j(t-\tau_j) + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{ij} u_{ij} + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{ij} \overline{u}_{d_q}$$

$$= A_{ij} x_i(t) + b_{ij} r_i(t) + (A_i - A_{ij}) e_i(t)$$

$$+ \sum_{q \neq j \neq \ldots \neq k \neq l} b_{ij} \Theta_{x_{ijq}}^* e_j(t) + A_{ij} e_i(t-\tau_i)$$

(12)

With the use of (3), such a choice of the parameters $\theta_{x_{iq}}^*$, $\theta_{uiq}^*$, and $\theta_{d}^*$ result in the following error equation

$$\dot{e}_i(t) = A_i + \sum_{q \neq j \neq \ldots \neq k \neq l} b_{iq} \Theta_{x_{ijq}}^* e_j(t) + + A_{ij} e_i(t-\tau_i).$$

(5)

The stabilizability assumption guarantees that there exists a constant $\theta_{x_{iq}}^*$ such that the control objective is fulfilled,
i.e. \( \lim_{t \to \infty} ||e(t)|| = ||x(t) - x(t)|| = 0. \)

To find the conditions when the equations in (11) can be satisfied, the marginal case - all but one actuator in the each local subsystem fail and the failure pattern is \( u_{ij}(t) = \bar{u}_j \), \( j = 1, \ldots, q - 1, q + 1, \ldots, m \) - is considered, as in (10).

If only the \( q \)th actuator does not fail we get from (11) \([13]\) \[
\dot{e}_i(t) = (A_i + b_q \Theta_{\text{oriq}}^T)e_i(t) + A_q e_i(t - \tau) - A_{ij} + b_q \Theta_{\text{oriq}}^T = 0, \quad b_q \Theta_{\text{oriq}} - b_{ji} = 0 \tag{15}\]
If we define the parameters \( \Theta_{\text{oriq}} \in \mathbb{R}^n \), \( \Theta_{\text{oriq}}^* \in \mathbb{R}^n \) and \( \Theta_{\text{oriq}}^* \in \mathbb{R}^n \) from the equations in (15), the stablizability assumption implies that there exist finite constant parameters \( \Theta_{\text{oriq}} \in \mathbb{R}^n \), in the control law (10),

\[
u_{pq}^*(t) = \Theta_{\text{oriq}}^T e_i(t) + \Theta_{\text{oriq}}^T x_i(t) + \Theta_{\text{oriq}}^T r_i(t), \tag{16}\]

such that perfect model following is achieved, i.e. \( \lim_{t \to \infty} e_i(t) \to 0 \) if \( t \to \infty \).

In summary, see the details in [12,15], the necessary and sufficient condition for the existence of actuator failure compensation in each subsystem is that there exist constant parameters \( \Theta_{\text{oriq}}^* \), \( \Theta_{\text{oriq}}^* \), and \( \Theta_{\text{oriq}}^* \) such that \( A_i - A_{ij} + b_q \Theta_{\text{oriq}}^T = 0 \), \( A_q + b_q \Theta_{\text{oriq}}^T = 0 \), \( b_q \Theta^* - b_{ji} = 0 \).

### 4.2 Basic tracking error equation

Introducing the parameter errors \( \tilde{\Theta}_{\text{oriq}}(t) = \Theta_{\text{oriq}}(t) - \Theta_{\text{oriq}}^* \), \( \tilde{\Theta}_{\text{oriq}}(t) = \Theta_{\text{oriq}}(t) - \Theta_{\text{oriq}}^* \), \( \tilde{\Theta}_{\text{oriq}}(t) = \Theta_{\text{oriq}}(t) - \Theta_{\text{oriq}}^* \), \( \tilde{\Theta}_{\text{oriq}}(t) = \Theta_{\text{oriq}}^*(t) - \Theta_{\text{oriq}}^* \), \( \tilde{\Theta}_{\text{oriq}}(t) = \Theta_{\text{oriq}}^*(t) - \Theta_{\text{oriq}}^* \), \( \tilde{\Theta}_{\text{oriq}}^*(t) = \Theta_{\text{oriq}}^*(t) - \Theta_{\text{oriq}}^* \) where \( \Theta_{\text{oriq}}(t), \Theta_{\text{oriq}}^*(t), \Theta_{\text{oriq}}^*(t), \Theta_{\text{oriq}}(t), \Theta_{\text{oriq}}^*(t), \) and \( \Theta_{\text{oriq}}(t) \) are the adaptation gains from (6), applying (4) to the actual plant (1), by assumption that there are \( g_i \) failed actuators and then using (3) and (11), we obtain

\[
\dot{e}_i(t) = A_i e_i(t) + \sum_{j=1, j \neq i}^m b_{ij} \Theta_{\text{oriq}}^T e_i(t - \tau) - A_{ij} + b_q \Theta_{\text{oriq}}^T = 0, \quad b_q \Theta^* - b_{ji} = 0. \tag{11}\]

Then with (7), using the value \( b_q = b_{ij} \Theta_{\text{oriq}}^* \) from (15) and the error \( e_i(t - \tau) = x_i(t - \tau) - x_i(t - \tau) \) we get

\[
\dot{e}_i(t) = A_i e_i(t) + \sum_{j=1}^m b_{ij} \Theta_{\text{oriq}}^* e_i(t - \tau) + \sum_{j=1}^m b_{ij} \Theta_{\text{oriq}}^* x_i(t - \tau) + \sum_{j=1}^m b_{ij} \Theta_{\text{oriq}}^* r_i(t) + \frac{\dot{\Theta}_{\text{oriq}}^*}{\tau}(t) + \frac{\dot{\Theta}_{\text{oriq}}^*}{\tau}(t) + \frac{\dot{\Theta}_{\text{oriq}}^*}{\tau}(t) \tag{17}\]

where \( a_{iq}^* \in \mathbb{R}^n \) is the \( q \)th row of \( A_q \) in (1).
Then we obtain the basic tracking error equation for stability analysis and adaptation algorithms design:

\[
\dot{e}_i(t) = A_i e_i(t) + \sum_{l=1}^{m} \hat{A}_{i(l)} \dot{a}_{i(l)} e_i(t - \tau_{l}) + \sum_{q=1}^{m} b_{q(i)} \Theta_{\alpha_{q}} \hat{\Theta}_{\alpha_{q}}(t) o_q(t) + \sum_{l=1}^{m} \sum_{l'=i}^{m} b_{l'} \Theta_{\alpha_{q}} \hat{\Theta}_{\alpha_{q}}(t) x_{l'}(t - \tau_{l'})
\]

(22)

V. ADAPTATION ALGORITHMS AND STABILITY ANALYSIS

To design the update laws for the control parameter matrices \( \Theta_{\alpha_{q}}(t) \) and \( \Theta_{\alpha_{q}}(t) \) in the adaptive control (21), we use the following Lyapunov-Krasovskii type functional

\[
V(\bullet) = \sum_{i=1}^{m} V_i = V_{el} + V_{qi} + \sum_{l=1}^{M_{l}} (V_{el} + V_{qi})
\]

where

\[
\begin{align*}
V_{el} &= \sum_{q=1}^{m} \left[ \Theta_{\alpha_{q}}^{-1} \left( \hat{\Theta}_{\alpha_{q}} \hat{\Theta}_{\alpha_{q}}(t) + \int_{t-\tau}^{t} \Theta_{\alpha_{q}}(s) \Gamma_{i(l)} \Theta_{\alpha_{q}}(s) ds \right) \right] \\
V_{qi} &= \sum_{q=1}^{m} \left[ \Theta_{\alpha_{q}}^{-1} \left( \hat{\Theta}_{\alpha_{q}} \Gamma_{i(l)} \hat{\Theta}_{\alpha_{q}}(s) + \int_{t-\tau}^{t} \Theta_{\alpha_{q}}(s) \Gamma_{i(l)} \Theta_{\alpha_{q}}(s) ds \right) \right]
\end{align*}
\]

(23)

where

\[
\begin{align*}
\hat{\Theta}_{\alpha_{q}} &= \hat{\Theta}_{\alpha_{q}}(t) + \Theta_{\alpha_{q}}(t) + \Theta_{\alpha_{q}}(t - h_i), \\
\hat{\Theta}_{\alpha_{q}} &= \hat{\Theta}_{\alpha_{q}}(t) + \Theta_{\alpha_{q}}(t) + \Theta_{\alpha_{q}}(t - h_i)
\end{align*}
\]

(24)

and \( Q = Q^T > 0 \), \( \Gamma_{i(l)} = \Gamma_{i(l)} > 0 \), and \( \Gamma_{i(l)} = \Gamma_{i(l)} > 0 \) are matrices of corresponding dimensions. The matrix \( P_i = P_i^T > 0 \) is computed from the Lyapunov equation

\[
A_i^T P_i + P_i A_i + Q_i + Q_i = 0, \quad Q_i = Q_i^T > 0
\]

(25)

and

\[
\eta_{iq} = \frac{1}{r_0} \Theta_{\alpha_{q}} [b_{q(i)} P_i 0 0 0] \quad r_0 > 0
\]

(26)

where \( r_0 > 0 \) is some scalar constant. This constant \( r_0 \) and the time-varying vectors \( \eta_{iq}(t) \) and \( \eta_{iq}(t) \) are “artificial” whose values will be defined later. These parameters are used only in the process of the stability proof. The parameters \( h_i \) and \( h_i \) are design parameters in the delayed components of the adaptation algorithms, as will be seen below.

We now choose the adaptation algorithms as

\[
\begin{align*}
\hat{\Theta}_{\alpha_{q}} &= -\eta_{iq}(t) - \hat{\eta}_{iq}(t) - \eta_{iq}(t - h_i) \\
\eta_{iq}(t) &= \text{sign} \left[ \Theta_{\alpha_{q}} \Gamma_{i(l)} o_q(t) e_i^T(t) \right] P_i b_{q(i)} \\
\hat{\Theta}_{\alpha_{q}} &= -\eta_{iq}(t) - \eta_{iq}(t - h_i) \\
\eta_{iq}(t) &= \text{sign} \left[ \Theta_{\alpha_{q}} \Gamma_{i(l)} x_{l'}(t - \tau_{l'}) e_i^T(t) P_i b_{q(i)}
\end{align*}
\]

(27)

where \( h_i \) and \( h_i \) are some design parameters.

Remark 2. Although only the integral component \( \eta_{iq}(t) \) (\( \eta_{iq}(t) = 0 \) and \( \eta_{iq}(t - h_i) = 0 \) in (27)) of the adaptation algorithm is needed for stability and exact asymptotic tracking, the use of the proportional and the proportional delayed terms in the adaptation algorithm (27) makes it possible to achieve better adaptation performance than the traditional integral (I) and proportional integral (PI) schemes. See e.g. [15] in which a proportional integral time delay (PITD) adaptation algorithm is used for centralized adaptive control. The adaptation algorithm in (27) includes the traditional gains \( \Gamma_{i(l)} \) in (26).

Using (25), the time derivatives of the components of (23) along (22) can be written as

\[
\begin{align*}
\dot{V}_{el}(k+1) &= e_i^T(k) [A_i^T P_i + P_i A_i + Q_i + Q_i] e_i(k) \\
&+ 2 \dot{e}_i^T(k) \sum_{l=1}^{m} b_{q(i)} \Theta_{\alpha_{q}}^{-1} \Theta_{\alpha_{q}}(t) o_q(t) \\
&+ 2 \dot{e}_i^T(k) \sum_{q=1}^{m} b_{q(i)} \Theta_{\alpha_{q}}^{-1} \hat{\Theta}_{\alpha_{q}}(t) x_{l'}(t - \tau_{l'})
\end{align*}
\]

(28)

\[
\begin{align*}
\dot{V}_{qi}(k+1) &= \sum_{q=1}^{m} \left[ \Theta_{\alpha_{q}}^{-1} \left( \hat{\Theta}_{\alpha_{q}} \Gamma_{i(l)} \hat{\Theta}_{\alpha_{q}}(s) + \int_{t-\tau}^{t} \Theta_{\alpha_{q}}(s) \Gamma_{i(l)} \Theta_{\alpha_{q}}(s) ds \right) \right] \\
&+ 2 \sum_{q=1}^{m} \left[ \Theta_{\alpha_{q}}^{-1} \left( \eta_{iq}(t) + \eta_{iq}(t - h_i) \right)^T \Gamma_{i(l)} \eta_{iq}
\end{align*}
\]

(29)

Combining (27) and (30) it follows that

\[
\begin{align*}
\dot{V}_{\eta_{iq}}(k+1) &= 2 \sum_{q=1}^{m} \left[ \Theta_{\alpha_{q}}^{-1} \left( \eta_{iq}(t) + \eta_{iq}(t - h_i) \right)^T \Gamma_{i(l)} \eta_{iq}
\end{align*}
\]

(30)
\[
\dot{V}_{y_1} \leq -r_0 \sum_{q=1}^{m} e_i(t)^T P_i b_i \Theta_{orig}^{-1} \Theta_{orig} \Gamma_{iq}^{-1} (\eta_{iq}(t) + \eta_{iq}(t-h_i)) \quad (32)
\]

Similarly we have
\[
\dot{V}_{y_2} \leq \frac{1}{M} \sum_{i=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) + \beta \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T P_i b_i \Theta_{orig}^{-1} \Theta_{orig} \Gamma_{i\bar{q}}^{-1} (\eta_{i\bar{q}}(t) + \eta_{i\bar{q}}(t-h_i)) \quad (33)
\]

Then using (25), (32), and (33) we get
\[
\dot{V}_{y_{22}} \leq \frac{1}{M} \sum_{i=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) + \beta \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T P_i b_i \Theta_{orig}^{-1} \Theta_{orig} \Gamma_{i\bar{q}}^{-1} (\eta_{i\bar{q}}(t) + \eta_{i\bar{q}}(t-h_i)) \quad (34)
\]

After completing the squares we have
\[
\dot{V}_{y_{22}} \leq -e_i(t)^T \tilde{Q}_i e_i(t) + \frac{1}{M} \sum_{i=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) + \frac{1}{M} \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t-h_i) + \frac{1}{M} \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) + \frac{1}{M} \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) \quad (35)
\]

where \( \Psi_{i\bar{q}} = \sum_{q=1}^{m} \frac{M-1}{2} a_i a_{i\bar{q}} a_{i\bar{q}}^T \).

Let us define the matrix \( \tilde{Q}_i = Q_{s1} + Q_{s2} \) such that \( Q_{s1} = Q^T_i \) and \( Q_{s2} = Q^T_{i\bar{q}} \). If we select the value \( r_0 \) from the inequality \( r_0 > \lambda_{min}(\Psi_{i\bar{q}}) \), where \( \lambda_{min}(\cdot) \) and \( \lambda_{max}(\cdot) \) are the maximum and minimum eigenvalues of \( \cdot \), we obtain from (35)

\[
\dot{V}_{y_{22}} \leq -e_i(t)^T \tilde{Q}_i e_i(t) + \frac{1}{M} \sum_{i=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t) + \frac{1}{M} \sum_{i=1}^{m} \sum_{q=1}^{m} e_i(t)^T \tilde{Q}_i e_i(t-h_i) \quad (36)
\]

for \( t \in (T_r, T_{r+1}), \) i.e. \( \dot{V}_{y_{22}} \) is negative semi-definite. Following [22], we have hence proved that the adaptive control (7) and the update algorithms (27) guarantee that \( V(t) \) and, therefore, \( e_i(t) \), \( \tilde{\Theta}_{orig} \in L_\infty \). The remainder of the stability analysis follows directly using the steps in [23].

We have thus obtained the following result.

**Theorem 1.** Consider system (1) and the reference model (3). Then the adaptive control (4), (21) with update laws (27) and actuator failures model (2) assures that the closed loop signals are bounded and that the tracking error \( e(t) = [e_1(t), \ldots, e_d(t)]^T \) converges to zero asymptotically.

**Remark 3.** We note that the coefficient matrices \( Q_1, Q_2 \) and the scalar \( r_0 \) are used only for analysis and do not influence the control law. Decentralized controller gains adjust automatically to counter the non-desirable effects of delayed interconnections, actuator failures and parameter uncertainties.

**VI. SIMULATION**

To illustrate the application of the proposed adaptive scheme, let us consider a plant with two subsystems described as

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u_0(t)
\begin{bmatrix}
0 & 0 \\
-1 & -2
\end{bmatrix} x_2(t-\tau_1) +
\begin{bmatrix}
0 & 0 \\
1 & 3
\end{bmatrix} u_0(t) \quad (37)
\]

We choose the reference model as

\[
\begin{bmatrix}
\dot{x}_{r1}(t) \\
\dot{x}_{r2}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-1 & -3
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} r(t)
\begin{bmatrix}
0 & 0 \\
-1 & -2
\end{bmatrix} x_2(t) +
\begin{bmatrix}
0 & 0 \\
1 & 3
\end{bmatrix} r(t) \in \mathbb{R}^2.
\]
\[ x_{r1}(0) = [-0.5 \ 0.5]^T, \quad x_{r2}(0) = [0.5 \ -0.5]^T \] (38)

The input signals of the reference models \( r_1 \) and \( r_2 \) are \( r_1 = r_2 = 2\sin(t) \). All parameters except the time delays \( \tau_1 = 5, \tau_{12} = 5, \tau_2 = 4, \tau_{21} = 4 \) are unknown to the controller.

The adaptation algorithms (27) in our simulation are

\[
\theta_{q_i}(t) = -PITD(\omega_i(t) e_i^T(t) P_{i l}, i, q, l = 1, 2; l \neq i)
\] (39)

where \( PITD(*) \) is the operator form for \( PITD(Z_i(t)) = \kappa_i \int_0^t Z_i(s) ds + k_P Z_i(t) + k_D Z_i(t - \Delta) \), where the parameter values were chosen as \( h_1 = h_2 = 1, \kappa_i = 1, k_P = 0.5, k_D = 0.05 \), with \( \omega_i = [e_i(t) x_i(t) x_i(t - \tau_i) r_i(t) l x_i(t - \tau_{il})]^T \), and \( P_i \) from (25)

\[
P_1 = \begin{bmatrix} 9.1667 & 2.5000 \\ 2.5000 & 1.6667 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 7.5000 & 2.5000 \\ 2.5000 & 5.0000 \end{bmatrix}
\]

In the simulation study, we suppose that the first control input of the subsystem 1 fails at \( t = 30 \) seconds and the first control input of the subsystem 2 fails at \( t = 50 \) seconds. Some simulation results are found in Fig. 1.

All simulation results verified the desired system performance. At the time instant when one of the actuators fails, there is a transient system response in the tracking error, and as time goes on, the tracking errors starting from a transient value, converge to zero, i.e. asymptotic tracking is achieved despite the unknown actuators failures and unknown system parameters. The values of controller parameters \( \theta \) also jump when one actuator failure occurs, and then converge to constant values.

VII. CONCLUSIONS

We have developed a coordinated adaptive decentralized control scheme for state delayed systems with unknown actuator failures. This scheme ensures asymptotic exact tracking in the presence of unknown plant parameters and unknown actuator failure parameters. Simulation results verified the desired performance of the developed coordinated adaptive decentralized controller. For the updating of the decentralized controller parameters, we develop a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes. A special Lyapunov-Krasovskii functional is introduced to design the update mechanism for the controller parameters and prove stability. The matching condition with \( B_i \) for the interaction terms in (1) is restrictive, but to the best of our knowledge, common to many decentralized MRAC of plants with delay within the framework of the direct adaptive approach, without actuator failure. The presence of actuator failures further complicates the problem. We found one way to design, and our paper, with its restrictive assumption, is a first important step. Our numerous simulations show, however, only a very slight sensitivity of performance to a breach of the matching condition. These results gives us the hope that this restriction is technical only, needed for the present proof of the closed loop system stability. We are currently working to develop a suitable proof technique without it.
REFERENCES


