SLIDING MODE CONTROL FOR INVERTIBLE SYSTEMS BASED ON A DIRECT DESIGN OF INTERACTORS

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ABSTRACT

The sliding mode tracking control is reviewed and extended so that it can be applicable to general invertible systems. The main advantage of this paper is a direct and numerical stable design of interactors for general invertible systems which is performed by calculating infinite eigenbasis of system matrices.

KeyWords: Sliding mode control, singular system, interactor, zeros.

I. INTRODUCTION

The sliding mode control (SMC) is well known as a robust control methodology which is applicable to certain nonlinear control systems and has been applied to many industrial control problems [1].

When the SMC is applied to a system basically described by \( \dot{x} = Ax + Bu \), a bang-bang control involving the switching function: \( \sigma = Cx \) is used. In order to guarantee occurrence of sliding mode, we usually assume that \( CB \) is non-singular [2]. And, the stability of sliding mode is ensured if and only if the invariant zeros of the system \( (C, A, B) \) have negative real parts [1,2,3,5,7].

When we design a regulator, we can choose \( C \) to satisfy these two conditions. However, when we design a tracking control system, \( \sigma \) is needed to coincide with the control error \( e \) for a given controlled output \( y \), which means that we cannot choose \( C \) freely. In this case, \( CB \) no longer is non-singular in general.

If the system \( (A, B, C) \) satisfies the decoupling condition [4], it is easy to get around this problem like the SMC for robotic manipulators [5,7]. Verghese et al. [3] used the interactor to get around this problem and extended \( C \) to the extent that the plant is invertible and propose a sliding mode control for invertible systems which requires measurement of state variables as well as derivatives of command signals. As for nonlinear systems, [15] solved a similar problem for the flat system.

In this paper, we first review SMC for the linear systems and will derive SMC for the invertible systems by a different way from that of Verghese et. al. [3]. We will introduce a direct design method of interactors for invertible transfer functions using infinite eigenvectors of system matrices.

As far as interactors concerned, we know the structure algorithm proposed by Silverman [12], or the geometrical approach [13]. However, a closed as well as numerically stable design method to derive interactors has not been shown.

Verghese et. al. [3] just applied the outcome of the structure algorithm to the sliding mode control and designed the desired interactor by two steps.

Comparing to their results, our algorithm provides a direct and explicit expression of the interactor needed for the sliding mode control which can be designed by numerically stable computation as well [14].

II. SMC FOR INVERTIBLE SYSTEMS

2.1 Review of the conventional SMC [1,2,3,5,7]

The plant considered in this paper is given by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Bg(x,t) \\
y(t) &= Cx(t) \\
A &\in \mathbb{R}^{nxn}, B \in \mathbb{R}^{nxp}, C \in \mathbb{R}^{np}\end{align*}
\]  

(1)

where \( x \in \mathbb{R}^n \) is the state variable which is assumed to be measurable; \( u \in \mathbb{R}^p \) is the control input; \( y \in \mathbb{R}^p \) is the controlled output which has to follow a given command signal \( r(t) \in \mathbb{R}^p \); \( g(x,t) \in \mathbb{R}^p \) is an unknown nonlinear perturbation which satisfies

\[
\|g(x,t)\| \leq M(x,t)
\]  

(2)