MIXED CONSTRAINED INFINITE HORIZON LINEAR QUADRATIC OPTIMAL CONTROL

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ABSTRACT

For a given initial state, a constrained infinite horizon linear quadratic optimal control problem can be reduced to a finite dimensional problem [12]. To find a conservative estimate of the size of the reduced problem, the existing algorithms require the on-line solutions of quadratic programs [10] or a linear program [2]. In this paper, we first show based on the Lyapunov theorem that the closed-loop system with a mixed constrained infinite horizon linear quadratic optimal control is exponentially stable on proper sets. Then the exponentially converging envelop of the closed-loop trajectory that can be computed off-line is employed to obtain a finite dimensional quadratic program equivalent to the mixed constrained infinite horizon linear quadratic optimal control problem without any on-line optimization. The example considered in [2] showed that the proposed algorithm identifies less conservative size estimate of the reduced problem with much less computation.

KeyWords: Linear quadratic optimal control, constrained systems, linear discrete time systems.

I. INTRODUCTION

In 1960, employing state space approach, Kalman provided a complete Riccati equation based solution strategy for infinite horizon linear quadratic optimal control problems [8]. However, constraints are always present in any practical control problems. For instance, physical restriction of actuator limits the value the input can assume. Moreover due to safety, environmental regulation and so on, states of plant are desired to lie within a designated area in state space. Under presence of these constraints, closed-loop system becomes nonlinear and the analysis by Kalman is no longer valid. However due to difficulty associated with the nonlinearity, constrained infinite horizon linear quadratic optimal control problems remained unsolved until Sznaier and Damborg [12] pioneered the area in 1987 by realizing the fact that a constrained infinite horizon linear quadratic optimal control problem can be reduced to a finite dimensional quadratic program. This reduction was possible because, for the infinite horizon quadratic cost to be finite, the constraints are only active over a finite time horizon and thus the remaining infinite horizon unconstrained part of the problem beyond the finite time horizon can be analytically solved by the Kalman’s technique. Based on this fact, they proposed a solution strategy with a set of on-line quadratic programs. Their results were further extended and refined by Scokaert and Rawlings [10]. Recently, Chmielewski and Manousiouthakis provided a computationally less demanding technique where only an on-line linear program is required to find a finite dimensional quadratic program equivalent to the constrained infinite horizon linear quadratic optimal control problem [2].

In this paper, exponential stability of mixed constrained linear quadratic optimal control on proper sets is first established based on the Lyapunov theory. Exponential stability for all feasible trajectory is also claimed in [2] and [12] from the fact that each feasible trajectory converges exponentially. However, as shown in [4], exponential stability for all feasible trajectories cannot be achieved with any control strategy if the plant is marginal or unstable. From the exponential stability proofs, we can find an exponentially converging envelop associated with the exponential stability on proper sets. Since the envelop is exponentially converging to zero, we can