SLIDING MODE CONTROL SYNTHESIS OF UNCERTAIN
TIME-DELAY SYSTEMS

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ABSTRACT

Sliding mode control synthesis is developed for a class of uncertain time-delay systems with nonlinear disturbances and unknown delay values whose unperturbed dynamics is linear. The synthesis is based on a new delay-dependent stability criterion. The controller constructed proves to be robust against sufficiently small delay variations and external disturbances. An admissible upperbound such that the corresponding closedloop system remains globally asymptotically stable for each delay value less than this upperbound is derived. Performance issues of the controller are illustrated in a simulation study.

KeyWords: Sliding mode, time-delay system, robustness.

I. INTRODUCTION

The primary concern of the paper is robust control of uncertain time-delay systems within the framework of sliding mode control methods. The sliding mode control paradigm implies the deliberate introduction of so-called sliding motions (i.e., the motions along a manifold where the control signal undergoes discontinuities) into the control system and it consists of two steps [23]. First, a manifold, such that if confined to this manifold the system has desired dynamic properties, is designed. Second, a discontinuous control law, which drives the system to this manifold in finite time, is synthesized.

The sliding mode control strategy is to construct a feedback that guarantees a Lyapunov function, selected for a nominal system, to remain negative on the trajectories of the perturbed system. The controller, thus constructed, asymptotically stabilizes the system and since the motion along the manifold proves to be uncorrupted by matched disturbances, the closed-loop system is additionally guaranteed to have strong robustness properties against matched disturbances. Due to these advantages and simplicity of implementation, sliding mode controllers have widely been used in various applications [24].

Motivated by technological advances, the interest recently emerged in extending the sliding mode control approach to infinite-dimensional dynamic systems such as distributed parameter systems and time-delay systems. The earlier works [19, 27] on extensions of sliding mode control algorithms to infinite-dimensional systems ran into a major difficulty, caused by the presence of an unbounded infinitesimal operator in the plant equation, and called for further theoretical investigations. Presently, the sliding mode control synthesis in the infinite-dimensional setting is well documented [14,15,17,18]. This synthesis retains robustness features, similar to those possessed by its counterpart in the finite-dimensional case, and being complementary to the $H_{\infty}$-design, it constitutes a more practical approach to infinite-dimensional systems than the ones of high computational complexity outlined in [3,6,8,11].

The existing results [1,7,9,10,20,25] on application of sliding mode control algorithms to time-delay systems, governed by functional differential equations (FDE), have corroborated their utility for this class of systems as well. However, these results, being inherited from the finite-dimensional treatment, contained no arguments supporting the well-posedness of FDE solutions to substantiate the stability analysis. Even the solution concept for discontinuous time-delay systems has not yet been addressed.

Resolving the fundamental issue on the precise meaning of FDE with discontinuous right-hand side constitutes one of the contributions of the present paper. Substantial stability analysis and sliding mode control synthesis of time-delay systems with small time lags and