FEEDBACK STABILIZATION OF NONHOLONOMIC CONTROL SYSTEMS WITH DRIFT

Fazal-ur-Rehman

ABSTRACT

This paper presents a differential geometric approach for feedback stabilization of nonholonomic control systems with drift and its applicability is tested on two different systems possessing different algebraic structures: a system with six state variables and three controls, and a knife edge example. The approach is universal in the sense that it is independent of the vector fields determining the motion of the system, or of the choice of a Lyapunov function. The proposed feedback law is as a composition of a standard stabilizing feedback control for a Lie bracket extension of the original system and a periodic continuation of a specific solution to an open loop control problem stated for an abstract equation on a Lie group, an equation which describes the evolution of flows of both the original and extended systems. The open loop problem is solved as a trajectory interception problem in logarithmic coordinates of flows.

KeyWords: Feedback stabilization, systems with drift, logarithmic coordinates, Lyapunov function.

I. INTRODUCTION

The feedback synthesis method presented in this paper applies to systems of the type

\[ \dot{x} = g_0(x) + \sum_{i=1}^{m} g_i(x) u_i, \quad x \in \mathbb{R}^n \]

where \( g_i, i = 1, ..., m, m < n, \) are linearly independent, analytic vector fields on \( \mathbb{R}^n, g_0(x) \) is drift vector field on \( \mathbb{R}^n \) satisfying the condition \( g_0(0) = 0 \) and \( u_i \) are Lebesgue integrable control functions on the interval \([0, \infty)\). These types of systems are of practical interest as they often represent models of mechanical systems with non-integrable velocity constraints, known as nonholonomic systems. It is well known, see [2], that such systems cannot be stabilized by smooth or even continuous static state-feedback laws and that the dependence of the stabilizing control on time is essential, see [4]. Constructive synthesis methods have been presented, see e.g. [7-9] and [10], but rely heavily either: on the existence of suitable time-varying Lyapunov functions whose decrease to zero guarantees stabilization, or else on the existence of state space transformations which bring the system to a special form (for example a power or chained form).

A different approach to stabilizing feedback synthesis is proposed which employs the differential geometric techniques of [5] based on considering of what is known as the Lie bracket extension of the original system (1). An arbitrary Lyapunov function is first employed to furnish a closed loop stabilizing control for the extended system. The stabilizing time-invariant feedback control for the extended system is then combined with a periodic continuation of a solution to an open loop, finite horizon control problem stated in terms of a formal equation on an associated Lie group. The solutions of this formal equation correspond to flows of the original or extended systems and are solved in terms of logarithmic coordinates for flows, (for a definition see [12]). The purpose of the formal problem is to generate open loop controls such that the trajectories of the controlled extended system and the original (open loop) system intersect with a given frequency \( 1/T \).

The contribution of the paper can be summarized as follows:

- A new strategy is presented for feedback stabilization of nonholonomic control systems with drift and its ef-