STRUCTURAL DESIGN OF COMPOSITE NONLINEAR FEEDBACK CONTROL FOR LINEAR SYSTEMS WITH ACTUATOR CONSTRAINT

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ABSTRACT

The performance of the composite nonlinear feedback (CNF) control law relies on the selection of the linear feedback gain and the nonlinear function. However, it is a tough task to select an appropriate linear feedback gain and appropriate parameters of the nonlinear function because the general design procedure of CNF control just gives some simple guidelines for the selections. This paper proposes an operational design procedure based on the structural decomposition of the linear systems with input saturation. The linear feedback gain is constructed by two linear gains which are designed independently to stabilize the unstable zero dynamics part and the pure integration part of the system respectively. By investigating the influence of these two linear gains on transient performance, it is flexible and efficient to design a satisfactory linear feedback gain for the CNF control law. Moreover, the parameters of the nonlinear function are tuned automatically by solving a minimization problem. The proposed design procedure is illustrated by applying it to design a tracking control law for the inverted pendulum on a cart system.

Key Words: Composite nonlinear feedback, tracking, structural decomposition, input saturation.

I. INTRODUCTION

The composite nonlinear feedback (CNF) control technique is to improve the transient performance of the closed loop system by introducing a nonlinear feedback law. Nonlinear techniques for improving transient performance of servomechanisms can be tracked back to the work of McDonald [1], where an analytical interpretation of the effect of nonlinear elements for nonlinear servo problems is given by using phase plane and space analysis. The composite nonlinear feedback control was proposed by Lin et al. in [2] for a class of second order linear systems with input saturation in 1998. Turner et al. [3] extended the results of [2] to multivariable systems. Furthermore, Chen et al. [4] developed a CNF control to a more general class of systems with measurement feedback. The results of [4] are extended to multivariable systems in [5] and [6]. More recently, Lan et al. [7] extended the CNF control technique to a class of nonlinear systems. The CNF control technique for discrete-time systems can be found in [8] and [9]. The applications of the CNF control technique are also reported in the literature, for examples, the helicopter flight control systems [10] and the hard disk drive servo systems [4, 11].

A CNF control law consists of a linear feedback control and a nonlinear feedback control. The philosophy of CNF control is that the linear part is designed to
yield a closed-loop system with a small damping ratio for a quick response, and the nonlinear part is introduced to increase the damping ratio of the closed-loop system while the system output approaches the reference target to reduce the overshoot caused by the linear part. Thus, there are two key issues in the design of the CNF control law. First, how to design the linear feedback gain? Second, how to select an appropriate nonlinear function? In [12], we introduced a scaled nonlinear function to improve the performance robustness to the variation of the tracking targets. It is also shown in [12] that, once the linear feedback gain and the form of the nonlinear function are selected, the parameters of the nonlinear function can be tuned automatically by solving a minimization problem. However, how to select the linear feedback gain is seldom addressed in the literature. In the design procedure proposed in [4] and [5], some simple guidelines are mentioned on the design of the linear feedback gain. But, the guidelines are not operational in the design of the linear feedback gains. In this paper, we apply the special coordinate basis (SCB) technique [13, 14] to the linear systems with input saturation. It is shown that the structural decomposition of the linear systems with input saturation has the same structural system matrices as the linear systems without input saturation, except that the saturation level in the SCB coordinate is different from the one in the original coordinate. The systems in the SCB form can be divided into three subsystems: stable zero dynamics, unstable zero dynamics, and integration subsystem. Based on the structural decomposition of the system, a linear feedback gain is constructed by two linear gains. One linear gain is designed to stabilize the unstable zero dynamics, the other one is selected to stabilize a pure integration system. Since the system in the SCB coordinate has a clear map among input, output and subsystems, it is not difficult for a control engineer to select these linear gains to construct the desired linear feedback gain for the CNF control law. On the other hand, the selections of these linear gains are independent. It is convenient to investigate the influence of these linear gains on the transient performance by simulations. Based on that, it is not difficult to obtain a satisfactory linear feedback gain for the CNF control law.

The paper is organized as follows: Section II formulates a tracking control problem and gives some introduction to the background of the CNF control. Section III presents the structural decomposition of the linear systems with input saturation. A novel design procedure of CNF control is proposed for the systems in SCB form in Section IV. Section V illustrates the design procedure by solving a tracking control problem for the invented pendulum on a cart system. Finally, Section VI gives some concluding remarks.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a linear system with input saturation

\[
\dot{x} = Ax + B \text{sat}(u), \quad x(0) = x_0
\]

\[
y = Cx
\]

where \( x \in \mathbb{R}^n \) is the state, \( u \in \mathbb{R} \) control input, \( y \in \mathbb{R} \) controlled output. \( A, B, C \) are constant matrixes with appropriate dimension, and sat: \( \mathbb{R} \rightarrow \mathbb{R} \) represents the actuator saturation defined as

\[
\text{sat}(u) = \text{sgn}(u) \min\{u_{\text{max}}, |u|\}
\]

with \( u_{\text{max}} \) being the saturation level of the input. The following assumptions on the system matrixes are required:

**A1.** \((A, B)\) is stabilizable;

**A2.** The system \((A, B, C)\) is invertible and has no zeros at \( s = 0 \).

We aim to design a composite nonlinear feedback (CNF) control law in the form of

\[
u = FX + Gr + \rho(r, y)B^T P(x - x_e)
\]

for the system (1)–(2) such that the resulting closed-loop system is stable and the output of the closed-loop system will track a step reference input \( r \) rapidly without experiencing large overshoot.

**Remark II.1.** A general design procedure of CNF control is proposed in [4] for a single-input single-output linear system, which gives a CNF control law of the form (4). Specifically, the linear feedback gain \( F \) is selected such that \( A + BF \) is stable, the feedforward gain \( G \) is given by

\[
G = -[C(A + BF)^{-1}B]^{-1}
\]

and

\[
x_e = -(A + BF)^{-1} BGr
\]

\( P \) is a positive definite solution of

\[(A + BF)^TP + P(A + BF) = -W\]

for some given \( W > 0 \). In general, we can simply let \( W = I \). For any \( \delta \in (0, 1) \), let \( c_\delta > 0 \) be the largest positive scalar satisfying the following condition:

\[
|FX| \leq u_{\text{max}}(1 - \delta), \quad \forall x \in X_\delta := \{x: x^TPx \leq c_\delta\}
\]

It is shown in [4] that, for any nonpositive function \( \rho(r, y) \), locally Lipschitz in \( y \), the composite nonlinear
feedback law in (4) is capable of driving the system controlled output \( y(t) \) to track asymptotically the step command input of amplitude \( r \), provided that the initial state \( x_0 \) and \( r \) satisfy

\[
\bar{x} := (x_0 - x_0) \in X_\delta, \quad |Hr| \leq \delta u_{max}
\]

where

\[
H = [1 - F(A + BF)^{-1}B]G
\]

Remark II.2. Various forms of the nonlinear function \( \rho(r, y) \) in (4) are used in the literature. In [12], a nonlinear function is given in the form of

\[
\rho(r, y) = -\beta e^{-\alpha y_0|y - r|}
\]

where

\[
x_0 = \begin{cases} \frac{1}{|y_0 - r|}, & y_0 \neq r \\ 1, & y_0 = r \end{cases}
\]

\( y_0 \) is the initial value of the output \( y \), and \( \alpha \) and \( \beta \) are the parameters to be tuned. Once the linear feedback gain \( F \) and the form of the nonlinear function \( \rho(r, y) \) are fixed, the parameters \( \alpha \) and \( \beta \) can be tuned automatically by solving the following minimization problem

\[
\min_{\alpha > 0, \beta > 0} I(e)
\]

where \( e = y - r \), and \( I(e) \) is the integral of absolute value of error (IAE), or the integral of time-multiplied absolute value of error (ITAE) criterion [15], that is, \( I(e) = \int_0^\infty |e|dt \) or \( I(e) = \int_0^\infty t|e|dt \)

However, how to select an appropriate linear feedback gain \( F \) is not addressed yet. This paper proposes a novel design procedure based on structural decomposition of the system. Under the proposed design procedure, the designer can select the linear feedback gain \( F \) with the help of structure information of the system.

III. STRUCTURAL DECOMPOSITION

To utilize the structure information of the system in the CNF controller design, we need to investigate the structural decomposition of the linear systems with input saturation. To this end, let us first consider a single-input and single-output linear system without saturation

\[
\dot{x} = Ax + Bu
\]

\[
y = Cx
\]

It follows from [13] and [14] that there exist nonsingular state and input transformations

\[
x = \Gamma_s \bar{x}, \quad u = \Gamma_i \bar{u}
\]

with \( \Gamma_s \in \mathbb{R}^{n \times n} \) and \( \Gamma_i \in \mathbb{R} \) such that

\[
\bar{x} = \bar{A} \bar{x} + \bar{B} \bar{u}
\]

\[
y = \bar{C} \bar{x}
\]

where \( \bar{A} = \Gamma_s^{-1} A \Gamma_s \)

\[
\bar{B} = \Gamma_s^{-1} B \Gamma_i
\]

\[
\bar{C} = C \Gamma_s = [0 \ 0 \ C_d]
\]

Moreover, \( \lambda(A_{aa}^-) \) contains all the system invariant zeros in the open left half plane, \( \lambda(A_{aa}^+) \) all the system invariant zeros in the closed right half plane. The systems in the form of (12)–(13) are called in the special coordinate basis (SCB) form.

The transformations (11) cannot apply to the linear system with input saturation (1)–(2) directly because of the nonlinearity of the saturation function (3). However, consider the state transformation

\[
x = \Gamma_s \bar{x}
\]

we have

\[
\dot{\bar{x}} = \Gamma_s^{-1}(Ax + B \text{sat}(u))
\]

\[
= \Gamma_s^{-1} A \bar{x} + \Gamma_s^{-1} B \Gamma_i \bar{u}
\]

\[
= \bar{A} \bar{x} + \bar{B} \text{sat}(u)
\]

\[
y = C \bar{x} = C \Gamma_s \bar{x} = \bar{C} \bar{x}
\]
where $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are given by (14)-(16), and
\[
\text{sat}(u) = \Gamma_i^{-1}\text{sat}(u)
\]
By the definition of the saturation function (3),
\[
\text{sat}(u) = \begin{cases}
\Gamma_i^{-1}u, & |u| \leq u_{\text{max}} \\
\Gamma_i^{-1}\text{sgn}(u)u_{\text{max}}, & |u| > u_{\text{max}}
\end{cases}
\]
Letting
\[
u = \Gamma_i \bar{u}
\]
yields
\[
\text{sat}(u) = \begin{cases}
\bar{u}, & |\Gamma_i \bar{u}| \leq u_{\text{max}} \\
\text{sgn}(\Gamma_i \bar{u})\Gamma_i^{-1}u_{\text{max}}, & |\Gamma_i \bar{u}| > u_{\text{max}}
\end{cases}
\]
\[
= \begin{cases}
\bar{u}, & |\bar{u}| \leq \tilde{u}_{\text{max}} \\
\text{sgn}(\bar{u})\tilde{u}_{\text{max}}, & |\bar{u}| > \tilde{u}_{\text{max}}
\end{cases}
\]
where
\[
\tilde{u}_{\text{max}} = |\Gamma_i^{-1}|u_{\text{max}}
\]
Defining
\[
\text{sat}(\bar{u}) = \text{sgn}(\bar{u}) \min[\tilde{u}_{\text{max}}, |\bar{u}|]
\]
we have
\[
\text{sat}(u) = \text{sat}(\bar{u})
\]
Thus, it is clear that the state and input transformations (11) transform the system (1)-(2) into the following system in the special coordinate basis form
\[
\dot{x} = \tilde{A}\bar{x} + \tilde{B}\text{sat}(\bar{u})
\]
\[
y = \tilde{C}\bar{x}
\]
It should be noted that the saturation functions in (1) and (18) have different saturation levels which are defined by (3) and (17) respectively.

**Remark III.1.** It is shown in [13] and [14] that Assumption A1 implies that $(A_{aa}^+, L_{aa}^+)$ is stabilizable.

## IV. DESIGN PROCEDURE

Partition and denote $\bar{x}$ as follows,
\[
\bar{x} = \begin{bmatrix}
x_a^- \\
x_a^+ \\
x_d
\end{bmatrix}, \quad x_a^- \in \mathbb{R}^{n_a^-}, \quad x_a^+ \in \mathbb{R}^{n_a^+}, \quad x_d \in \mathbb{R}^{n_d}
\]
then we can rewrite the system (18)-(19) into the following form
\[
\dot{x}_a^- = A_{aa}^- x_a^- + L_{aa}^- y
\]
\[
\dot{x}_a^+ = A_{aa}^+ x_a^+ + L_{aa}^+ y
\]
\[
\dot{x}_d = A_dx_d + B_d(E_1\bar{x} + \text{sat}(\bar{u}))
\]
\[
y = C_d x_d
\]
where
\[
E_1 = [E_{da}^- E_{da}^+ E_{dd}]
\]
It is shown in Section III that the system (1)-(2) is equivalent to the system (20)-(23). In this section, we propose a design procedure to construct a CNF control law for the system (20)-(23) which is in the SCB form. We assume that the given system (1)-(2) satisfies Assumptions A1 and A2, so does the system (20)-(23). The CNF control law can be constructed by the following step-by-step procedure.

**Step 1.** Select a vector $F_d \in \mathbb{R}^{1 \times n_d}$ such that $A_{aa}^+ + L_{ad}^+ F_a$ is asymptotically stable. This is possible because $(A_{aa}^+, L_{ad})$ is stabilizable by Remark III.1. And, define
\[
T_{n_a} = \begin{bmatrix}
T_{n_a^-} & T_{n_a^+} & t_1 & \cdots & t_{n_d}
\end{bmatrix}
\]
\[
E_2 = [0 - F_d (A_{aa}^+)^{n_d} e_1 \cdots e_{n_d}]
\]
and
\[
\begin{bmatrix}
0_{1 \times (n_0^n + n_0^i + i - 1)} \\
1 \\
-F_aL_{ad}^+ \\
\vdots \\
-F_a(A_{aa}^+)^{n_d-i-1}L_{ad}^+
\end{bmatrix}
\]
\[
e_i = -F_a(A_{aa}^+)^{n_d-i-1}L_{ad}^+
\]
for \(i = 1, \ldots, n_d\).

**Step 2.** Select \(F_d\) such that \(A_d + B_d F_d\) is asymptotically stable. Then the linear feedback gain is given by
\[
\bar{u}_L = \bar{F} \bar{x} + \bar{G} r
\]
where
\[
\bar{F} = F_d[0 \ I_{n_d}]T - E_1 - E_2
\]
and
\[
\bar{G} = -[\bar{C}(\bar{A} + \bar{B} \bar{F})^{-1} \bar{B}]^{-1}
\]

**Step 3.** Given a positive-definite matrix \(W \in \mathbb{R}^{d \times d}\), solve the Lyapunov equation
\[
(\bar{A} + \bar{B} \bar{F})^T \bar{P} + \bar{P}(\bar{A} + \bar{B} \bar{F}) = -W
\]
for \(\bar{P} > 0\). We also let
\[
\bar{x}_e = -r(\bar{A} + \bar{B} \bar{F})^{-1} \bar{B} \bar{G}
\]
Then, the nonlinear feedback control law is given by
\[
\bar{u}_N = \bar{p}(r, y) \bar{B}^T \bar{P}(\bar{x} - \bar{x}_e)
\]
where \(\bar{p}(r, y)\) is any non-positive function locally Lipschitz in \(y\).

**Step 4.** The CNF control law is given by combining the linear and nonlinear feedback law derived in the previous steps,
\[
\bar{u} = \bar{F} \bar{x} + \bar{G} r + \bar{p}(r, y) \bar{B}^T \bar{P}(\bar{x} - \bar{x}_e)
\]

**Theorem IV.1.** Consider the closed-loop system consisting of (20)–(23) and (33). For any \(\delta \in (0, 1)\), let \(c_\delta > 0\) be the largest positive scalar such that for all \(x \in X_\delta\), where
\[
X_\delta := \{x : x^T P x \leq c_\delta, |\bar{F} x| \leq (1 - \delta) \bar{u}_{\max}\}
\]
Then, for any nonpositive functions \(\bar{p}(r, y)\), locally Lipschitz in \(y\), the composite nonlinear feedback control law (33) is capable of driving the system output \(y(t)\) to track the step command input of amplitude \(r\) asymptotically, provided that the initial state \(\bar{x}(0)\) and the step amplitude \(r\) satisfy
\[
(\bar{x}(0) - \bar{x}_e) \in X_\delta \quad \text{and} \quad |\bar{H} r| \leq \delta \bar{u}_{\max}
\]
where
\[
\bar{H} = [I - \bar{F}(\bar{A} + \bar{B} \bar{F})^{-1} \bar{B}] \bar{G}
\]

**Proof.** Consider the linear system
\[
\dot{\bar{x}} = (\bar{A} + \bar{B} \bar{F}) \bar{x}
\]
Letting
\[
\bar{x} = T \bar{x}
\]
yields
\[
\dot{\bar{x}} = \begin{bmatrix}
A_{aa}^- & L_{ad}^- F_a & L_{ad}^- C_d \\
0 & A_{aa}^+ + L_{ad}^+ F_a & L_{ad}^+ C_d \\
0 & 0 & A_d + B_d F_d
\end{bmatrix} \bar{x}
\]
Since all the matrices \(A_{aa}^-, A_{aa}^+ + L_{ad}^+ F_a\) and \(A_d + B_d F_d\) are stable, the system (34) is asymptotically stable. That is, the matrix \(\bar{A} + \bar{B} \bar{F}\) is also stable. The remainder of the proof can follow the same lines of the proof of Theorem 1 in [4].

**Remark IV.1.** The CNF control law (33) is designed in the SCB coordinate. By the state and input transformations (11), it is not difficult to obtain the CNF control law in the original coordinate. Specifically,
\[
u = \Gamma_1 \bar{u}
\]
where
\[
\Gamma_1 = \Gamma_1 \bar{F} \Gamma_s^{-1} x + \Gamma_1 \bar{G} r + \rho(r, y) \Gamma_1 \bar{B}^T \bar{P}(\Gamma_s^{-1} x - \bar{x}_e)
\]
Let
\[
F = \Gamma_1 \bar{F} \Gamma_s^{-1}, \quad G = \Gamma_1 \bar{G},
\]
\[
\rho(r, y) = \Gamma_1^2 \bar{p}(r, y), \quad P = \Gamma_s^{-T} \bar{P} \Gamma_s^{-1},
\]
\[
x_e = \Gamma_s \bar{x}_e
\]
we have
\[
u = F x + G r + \rho(r, y) B^T P (x - x_e)
\]
which is in the same form of (4).

**Remark IV.2.** From the design procedure we can see that once the linear feedback gain \(\bar{F}\) is selected, the other parameters \(\bar{G}, \bar{P}\) and \(\bar{x}_e\) can be calculated by (29), (30) and (31) respectively. And, the nonlinear function
can be tuned automatically by solving a minimization problem (9) as described in Remark II.2. Thus, it is very important to select an appropriate linear feedback gain $\bar{F}$ in the design of the CNF control law such that the closed loop system has best transient performance. Consider the closed-loop system consisting of (20)–(23) and the linear feedback control

$$\bar{u} = \bar{F} \tilde{x}$$

If the control input is not saturated, by letting

$$\tilde{x} = \begin{bmatrix} \tilde{x}_a^+ \\ \tilde{x}_a^- \\ \tilde{x}_d \end{bmatrix} = T \tilde{x}$$

the closed-loop system is given by

$$\begin{align*}
\dot{\tilde{x}}_a^- &= A_{aa} \tilde{x}_a^- + L_{ad} F_a \tilde{x}_a^+ + L_{ad} \tilde{x}_1 \\
\dot{\tilde{x}}_a^+ &= (A_{aa}^+ + L_{ad} F_a) \tilde{x}_a^+ + L_{ad} \tilde{x}_1 \\
\dot{\tilde{x}}_d &= (A_d + B_d F_d) \tilde{x}_d
\end{align*}$$

$$y = [0 \ F_a \ C_d] \tilde{x}$$

It is clear that $F_a$ is designed to stabilize the unstable zero dynamics of the system, and $F_d$ is to stabilize the integration part of the system. Since $F_a$ and $F_d$ are selected independently in the proposed design procedure, we can investigate the influence of $F_a$ and $F_d$ on the transient performance so that a satisfactory $\bar{F}$ is obtained.

V. TRACKING CONTROL OF INVERTED PENDULUM SYSTEM

To demonstrate the design procedure proposed in this paper, we will design a CNF control law for the tracking control problem of the inverted pendulum on a cart system.

The inverted pendulum on a cart system, shown in Fig. 1, is a well known unstable nonlinear system that can be found in many universities’ control labs. Let $M$ be the mass of the cart, $m$ the mass of the block on the pendulum, $l$ the length of the pendulum, $y$ the position of the cart, $\theta$ the angle of the pendulum makes with vertical, $g$ the acceleration due to gravity, $b$ the coefficient of viscous friction for motion of the cart, and $u$ the applied force, the state space model of the inverted pendulum on a cart system is given by [16, 17],

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{g(x)}(f(x, u) - mg \cos x_3 \sin x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{g(x)}((M + m)g \sin x_3 - f(x, u) \cos x_3)
\end{align*}$$

where

$$x_1 = y, \quad x_2 = \dot{y}, \quad x_3 = \theta, \quad x_4 = \dot{\theta}$$

and

$$f(x, u) = u + mlx_2^2 \sin x_3 - bx_2$$

$$g(x) = M + m(\sin x_3)^2$$

with

$$x = [x_1 \ x_2 \ x_3 \ x_4]^T$$

Assume the maximum control input is $\pm 20N$, the linearization model with input saturation is given by

$$\dot{x} = Ax + B \text{sat}(u)$$

$$y = Cx$$

with $u_{\text{max}} = 20$.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b}{M} & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b}{lM} & \frac{(M + m)g}{lM} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{lM} \end{bmatrix}$$

and

$$C = [1 \ 0 \ 0 \ 0]$$
The system (35) has two invariant zeros at
\[ \pm \sqrt{\frac{g}{l}} \]
Thus, the inverted pendulum on a cart system is a non-minimum phase system. The objective is to design a CNF control law such that the closed-loop system is stable, and the output of (35) will track a step reference \( r \) as quick as possible with a very small overshoot or without any overshoot.

Assume the parameters of the system are given by

\[
M = 1.278 \text{ kg}, \quad m = 0.051 \text{ kg}, \quad l = 0.325 \text{ m} \\
g = 9.8 \text{ m/sec}^2, \quad b = 12.98 \text{ kg/sec} 
\]

then, the state and input transformations

\[ x = \Gamma_s \bar{x}, \quad u = \Gamma_i \bar{u} \]
with

\[
\Gamma_s = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0.1792 & -0.1792 & -3.0769 & 0 \\
-0.9838 & -0.9838 & 0 & -3.0769
\end{bmatrix}
\]

\[ \Gamma_i = 1.3780 \]

will transform the system (35) into the SCB form,

\[
\begin{align*}
\dot{x}_a^- &= A_{aa}^- x_a^- + L_{ad}^- y \\
\dot{x}_a^+ &= A_{aa}^+ x_a^+ + L_{ad}^+ y \\
\dot{x}_d &= A_d x_d + B_d (E_1 \bar{x} + \text{sat}(\bar{u})) \\
y &= C_d x_d
\end{align*}
\] (36)

where

\[
A_{aa}^- = -5.4913, \quad L_{ad}^- = 47.1535 \\
A_{aa}^+ = 5.4913, \quad L_{ad}^+ = 47.1535
\]
and

\[
A_d = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}, \quad B_d = \begin{bmatrix}
0 \\
1
\end{bmatrix}, \quad C_d = [1 \ 0] \\
E_1 = [-0.0070 \ 0.0070 \ 0.1203 \ -10.1565]
\]

Let

\[ F_a = -0.5 \]

then \( A_{aa}^+ + L_{ad}^+ F_a = -18.0855 \) is stable. Calculating \( T \) and \( E_2 \) by (25) and (26) gives

\[
T = \begin{bmatrix}
1.0000 & 0 & 0 & 0 \\
0 & 1.0000 & 0 & 0 \\
0 & 0.5000 & 1.0000 & 0 \\
0 & 2.7456 & 23.5767 & 1.0000
\end{bmatrix}
\]

\[ E_2 = [0 \ 15.0769 \ 129.4658 \ 23.5767] \]

Then, select

\[ F_d = [-3.185 \ -0.7] \]

which places the eigenvalues of \( A_d + B_d F_d \) at \(-0.35 \pm 1.75i\). The CNF control law in the SCB coordinate is given by

\[ \bar{u} = \bar{F} \bar{x} + \bar{G} r + \bar{\rho}(r, y) \bar{P}(\bar{x} - \bar{x}_e) \] (37)

where \( \bar{F}, \bar{G}, \bar{P} \) and \( \bar{x}_e \) are calculated by (28), (29), (30) and (31) respectively, and \( \bar{\rho}(r, y) \) is given by (7), i.e.,

\[ \bar{\rho}(r, y) = -\beta e^{-2q_2 |y - r|} \]

Solving the minimization problem (9)

\[
\min_{x>0, \beta>0} \int_0^\infty t |y - r|
\]

yields

\[ x = 4, \quad \beta = 4.625 \]

According to Remark IV.1, the CNF control law (37) in SCB coordinate can be transformed back to the original coordinate,

\[ u = F \bar{x} + G r + \rho(r, y) B^T \bar{P}(x - x_e) \] (38)

where

\[
F = \Gamma_i \bar{F} \Gamma_s^{-1} \\
G = \Gamma_i \bar{G} = -13.4060 \\
P = \Gamma_s^{-T} \bar{P} \Gamma_s^{-1}
\]

\[
\begin{bmatrix}
159.1511 & 54.2989 & 118.2632 & 19.9472 \\
54.2989 & 52.3575 & 114.7426 & 20.8955 \\
118.2632 & 114.7426 & 255.6285 & 46.0354 \\
19.9472 & 20.8955 & 46.0354 & 8.3834
\end{bmatrix}
\]
\[
x_e = \Gamma_i \ddot{x}_e = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}
\]

and

\[\rho(r, y) = \Gamma^2 \hat{\rho}(r, y) = -1.6333 \beta e^{-x_2 |y - r|}\]

For comparison, we also design a linear control law by the ITAE (the integral of the time multiplied by the absolute values of the error) method [15]. The ITAE method obtains the desired transient response by placing the closed-loop pole location to minimize the same performance index as we used to tune the parameters \(\alpha\) and \(\beta\); that is,

\[
\min \int_0^\infty t |y - r| dt
\]

By letting the nominal cutoff frequency

\[\omega_0 = 3 \text{rad/sec}\]

the ITAE method gives a linear control law

\[u = Fx + Gr\] (39)

where

\[F = [3.4327 \ 16.0696 \ 26.3997 \ 3.6208]\]

\[G = -(C(A + BF)^{-1} B)^{-1} = -3.4327\]

The simulation results are shown in Fig. 2. The closed-loop system under the linear control law (39) has an overshoot 4.88%. But under the CNF control law (38), there is only 0.12% overshoot in the transient response of the closed-loop system. Also, the settling time under the CNF control law is much smaller than that under the linear control law. Both of the control laws are not saturated.

**VI. CONCLUSIONS**

The composite nonlinear feedback technique is an efficient tool to improve the transient performance of the closed-loop system. The performance of the CNF control law relies on the selection of the linear feedback gain \(F\) and the nonlinear function \(\rho(r, y)\). This paper proposes a novel design procedure of the composite nonlinear feedback control for single-input and single-output linear systems with input saturation.

The linear systems with input saturation is transformed into a special coordinate basis form by structural decomposition. Then the CNF control law is designed for the system in SCB form. In the proposed design procedure, the linear feedback gain \(F_a\) and \(F_d\) is constructed by two feedback gains \(F_a\) and \(F_d\). \(F_a\) is selected to stabilize the unstable zero dynamics, and \(F_d\) is selected to stabilize the pure integration system. The parameters of the nonlinear function \(\rho(r, y)\) are tuned by solving a minimization problem. For an experienced designer, it is not difficult to select the appropriate \(F_a\) and \(F_d\) from the structure information of the system. Moreover, since the selections of \(F_a\) and \(F_d\) are independent, the designer can further investigate how \(F_a\) and \(F_d\) influence the best achievable transient performance by simulation. Thus, it is more flexible and effective than the design procedure proposed in [4].
REFERENCES


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