ITERATIVE LEARNING CONTROL USING ADJOINT SYSTEMS AND STABLE INVERSION

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ABSTRACT

In this paper, we investigate iterative learning control (ILC) for non-minimum phase systems from a novel viewpoint. For non-minimum phase systems, the magnitude of a desired input obtained by ILC using forward-time updating and Silverman’s inversion are too large because of the influence of the unstable zeros. On the other hand, stable inversion constructs a bounded desired input by using non-causal inverse for non-minimum phase systems.

In this paper, we first clarify that ILC using an adjoint system achieves the desired input defined by stable inversion. Hence, ILC using an adjoint system is an effective method for the control of non-minimum phase systems with uncertainty. However, a useful convergence condition of ILC using an adjoint system was not achieved. Next, we develop a simple convergence condition in the frequency domain.

KeyWords: Iterative learning control, stable inversion, non-minimum phase system, adjoint system.

I. INTRODUCTION

Robot manipulators are often required to achieve exact tracking for a given desired output $y_d(t)$. However, it is difficult for conventional feedback control to achieve such a requirement. One of the methods for achieving exact tracking is feedforward control based on the classical inversion [4]. With this method, a desired input that exactly generates the desired output is easily constructed. However, this method presents two problems. First, for non-minimum phase systems, the desired input increases exponentially with the evolution of time because of the unstable zeros of the systems. Second, this method requires completely accurate information about the systems to be controlled.

In order to overcome the first problem, Devasia et al. proposed a method called stable inversion [5], which is applied to non-minimum phase systems such as flexible manipulators. Using a non-causal inverse, they construct a bounded desired input and state, which can achieve exact tracking for a given desired output. Stable inversion is useful for non-minimum phase systems because it can avoid the influence of the unstable zeros of the systems.

However, as when using classical inversion, completely accurate information about the systems to be controlled is required. If there are any uncertainties about the systems, this approach cannot be used.

On the other hand, Arimoto et al. [1] proposed iterative learning control (ILC) to overcome the second problem. ILC enables us to find a desired input over a finite time interval through repetition of the same tasks. Even if the system has some uncertainties, it is possible to achieve exact tracking. Many theoretical studies on ILC have been published. In this paper, we discuss the following two methods:

1. a method using output error and its derivative in forward-time [1,2]. (which we call generalized PID-type (GPID-type))
2. a method using output error in backward-time [3]. (which we call adjoint-type)

If GPID-type ILC is applied to a non-minimum phase system, the magnitude of the input sequence is too large because GPID-type is closely related to the classical inversion.

Hence, classical inversion, stable inversion and the GPID-type ILC are not applicable to non-minimum phase systems with uncertainties. Since adjoint-type ILC enables us to find a desired input through the repetition of trials as GPID-type ILC, it was expected to be related to the classical inversion. However, numerical experiments
revealed that there is a close relationship between adjoint-type ILC and stable inversion.

The remainder of this paper is organized as follows. First, we review ILC and stable inversion. Second, we clarify the relationship between adjoint-type ILC and stable inversion theoretically and present a convergence condition on the time domain. However it is difficult to judge this condition is satisfied or not. Third, we develop a simple convergence condition on the frequency domain. Finally, we offer some concluding remarks.

II. PREPARATION

In this paper, we consider the following linear time-invariant system.

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = cx(t)$$    \hspace{1cm} (1)

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are state, input, and output, respectively. $A$, $b$ and $c$ are matrix or vector with appropriate dimensions. We assume the following for (1).

1. Matrix $A$ has no eigenvalues on the open right half complex plane and the imaginary axis.
2. (1) is a non-minimum phase system.
3. (1) has no zeros on the imaginary axis.
4. The relative degree is $r$.

2.1 Iterative learning control

ILC enables us to find a desired input that generates the desired output exactly over a finite time interval $[T_0, T_f]$ through repetition of trials. The basic structure of ILC is shown in Fig. 1. A control input in the next trial is defined by the current control input and the error signal, that is,

$$u_{k+1} = \Sigma(u_k, e_k)$$ \hspace{1cm} (2)

where index $k$ is a trial number; $u_k$, $y_k$, and $e_k = y_k - y_d$ are input, output and error signal on the $k$-th trial, respectively. $U_{[T_0, T_f]}$ and $E_{[T_0, T_f]}$ are input and error function space defined on $[T_0, T_f]$, respectively, and $\Sigma(\cdot, \cdot)$ is the operator such that $\Sigma : U_{[T_0, T_f]} \times E_{[T_0, T_f]} \rightarrow U_{[T_0, T_f]}$

![Fig. 1. Iterative learning control scheme.](image)

The problem of ILC is to design update law (2) using partial information about the system. In this paper, we focus on two design methods, GPID-type and adjoint-type ILC. Therefore, we review how the update law (2) is designed for (1) with $x(T_0) = 0$.

[GPID-type] [2]

This method uses a causal update as follows:

$$\dot{\hat{p}}_d(t) = A\hat{p}_d(t) + b_1 e^{(t)}, \ \ p_d(T_0) = 0$$
$$q_d(t) = c\hat{p}_d(t) + d_1 e^{(t)}$$
$$u_{k+1}(t) = u_k(t) - q_k(t)$$ \hspace{1cm} (3)

where $e^{(t)}$ indicates $r$ times derivative of $e$. If system (1) satisfies the following inequality

$$\left| 1 - d' d \right| < 1, \ (d = cA^{r-1}b),$$

then it can track the desired output by update law (3) and (4). We can choose the dynamical part $\{A, b, c\}$ of (3) arbitrarily.

[adjoint-type] [3]

This method uses a non-causal update law.

The input-output mapping of (1) is defined

$$[\mathcal{S}u(t)] \triangleq \int_{T_0}^{t} ce^{(r-\tau)}b u(\tau) d\tau.$$ \hspace{1cm} (6)

$\mathcal{S}$ is decomposed into a known part ($\hat{\mathcal{S}}$) and an unknown part ($\mathcal{U}$) as $\mathcal{S} = \hat{\mathcal{S}} \mathcal{U}$. The update law is constructed by using adjoint operator $\hat{\mathcal{S}}^*$ of the known part $\hat{\mathcal{S}}$, namely

$$u_{k+1} = \hat{\mathcal{S}} \mathcal{U} [u_k - y_d].$$ \hspace{1cm} (7)

When the system matrices of the known part $\hat{\mathcal{S}}$ are denoted by $(\hat{A}, \hat{b}, \hat{c})$, the system that corresponds to $\hat{\mathcal{S}}^*$ is defined as follows:

$$\dot{\hat{p}}_d(t) = -\hat{A}^T p_d(t) - \hat{c}^T e_d(t), \ p_d(T_0) = 0$$
$$q_d(t) = \hat{b}^T p_d(t).$$ \hspace{1cm} (8)

The next control input is defined

$$u_{k+1}(t) = u_k(t) - \alpha q_d(t).$$ \hspace{1cm} (9)

If the unknown part satisfies the inequality

$$\langle u_{\eta}, \eta \rangle \geq \beta \| \eta \|^2, \ (\beta > 0)$$ \hspace{1cm} (10)
for all $\eta \in L_2[0, \infty]$, then tracking to the desired output can be achieved by update law (7) with $0 < \alpha < 2\beta \sqrt{\xi}$. The inner product and norm are defined as $\langle x, y \rangle = \int_0^{\tau_f} x^T y \, dt$ and $\|x\| = \sqrt{\langle x, x \rangle}$, respectively.

### 2.2 Stable inversion

A method called stable inversion was proposed in [5]. Even if the system to be controlled is non-minimum phase, stable inversion gives the bounded desired input which achieves exact tracking for a given desired output by using non-causal inversion. This method contrasts with the classical inversion proposed in [4]. A stable inversion problem for (1) is formulated as follows.

**[Stable Inversion Problem]**

Given a desired output $y_d(t) \in L_1 \cap L_\infty$, find an input $u_d$ and a state $x_d$ such that

1. $u_d(t)$ and $x_d(t)$ satisfy the differential equation
   \[
   \dot{x}_d(t) = Ax_d(t) + bu_d(t)
   \]
   (11)

2. $x_d$ is mapped exactly to the desired output $y_d(t)$,
   \[
   cx_d(t) = y_d(t), \quad \forall t \in (-\infty, \infty)
   \]
   (12)

3. $u_d, x_d \in L_1 \cap L_\infty$.

We call $x_d$ and $u_d$ the desired state and the desired input, respectively.

Stable inversion problem is solved as follows.

Since relative degree is $r$, we define $\xi_i = cA^{r-1}x_i (i = 1, \ldots, r)$, and denote $\xi = [\xi_1, \ldots, \xi_r]^T$. We choose $\eta = [\eta_1, \ldots, \eta_{r-1}]^T$ appropriately such that the state transformation
   \[
   [\xi^T, \eta^T]^T = \Lambda x
   \]
   (13)

is nonsingular. In the new coordinates, (1) becomes

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\vdots \\
\dot{\xi}_r &= \alpha_r [\xi^T, \eta^T]^T + \beta_r u \\
\eta &= \alpha_r [\xi^T, \eta^T]^T + \beta_r u
\end{align*}
\]
(14)

where $\alpha_r \in R^{r \times r}$, $\beta_r \in R$, $\alpha_r \in R^{(n-\gamma) \times r}$, and $\beta_r \in R^{(n-\gamma) \times 1}$. Since $\beta_r \neq 0$, the following equation

\[
\eta = Q \eta + g(t)
\]
(17)

is well defined. To maintain exact tracking we choose $\xi = y_d^{-1}(\Delta \xi_d)$ for $i = 1, \ldots, r$, and (16) is substituted into (15). Then, the following differential equation

\[
\eta = Q \eta + g(t)
\]
(17)

is obtained, where $g$ is constructed by linear combination of the desired output $y_d$ and its derivative $y_d'$. (1) becomes

\[
\dot{\xi}_i = \xi_{i+1}, \quad i = 1, \ldots, r
\]
(18)

where all eigenvalues of $Q$. ($Q_0$) exist in the open left (right) half complex plane. State transition matrix of the differential equation (17) is defined

\[
\Phi(t) = \begin{bmatrix}
\Phi(t)^0 \\
\Phi(t)^T
\end{bmatrix}
\]

and its derivative

\[
\Phi(t)^0 = \begin{bmatrix}
e^{\Phi(t)^0} X(t) & 0 \\
0 & e^{\Phi(t)^T} X(t)
\end{bmatrix}
\]
(19)

where

\[
X(t) = \begin{cases}
1 & t > 0 \\
0 & t < 0
\end{cases}
\]

The solution of (17) under the boundary conditions $\eta(\pm \infty) = 0$ is

\[
\eta_d(t) \Delta \eta(t) = \int_{-\infty}^{t} \Phi(t, \tau)g(\tau) \, d\tau.
\]
(20)

Since $g$ is constructed by $y_d$ and $\xi_d$, $g \in L_1 \cap L_\infty$. Clearly, $\eta_d \in L_1 \cap L_\infty$. The desired state $x_d$ is obtained by the inverse coordinate transformation of (13), and the desired input is obtained by (16).

**Remark 1.** In the classical inversion, the differential equation (17) is solved under the initial condition $\eta(0) = \eta_0$. Therefore, when (1) is the non-minimum phase system, the desired state and the input diverge as evolution of time.

**Remark 2.** Since (1) is a non-minimum phase system,
there exists $Q$. Even if the desired output has the bounded support $[t_0, t_f]$, the desired input $u_d$ has a nonzero value before $t_0$. This is the remarkable property of stable inversion.

**Example 1.** Consider the following non-minimum phase system

$$
\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -6 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)
$$

$$
y(t) = [-1 & 0 & 1] x(t), \quad x(0) = 0
$$

and the desired output given by

$$y_d(t) = \begin{cases} 1 - \cos t & 0 \leq t \leq 2\pi \\ 0 & \text{otherwise} \end{cases} \quad (22)
$$

When the trial interval is $[-2\pi, 3\pi]$, input and output of GPID-type ILC and classical inversion are shown in Figs. 2 and 3, respectively. Input and output of adjoint-type ILC and stable inversion are shown in Fig. 4 and 5, respectively. Furthermore, the inputs of GPID-type ILC and the desired input defined by classical inversion and stable inversion on $[-1, 2]$ are shown in Fig. 6.
III. A RELATIONSHIP BETWEEN ADJOINT-TYPE ILC AND STABLE INVERSION

When ILC is applied, input satisfying \( y_d = Su \) is obtained by repeating trials. Therefore, both GPID and adjoint-type ILC are expected to be closely related to the classical inversion. However, from the example in the previous section, it is conjectured that adjoint-type ILC is related to the stable inversion. In this section, we discuss this relationship theoretically. Because the stable inversion problem is defined on the infinite time interval \((-\infty, +\infty)\), we extend the trial interval of ILC on the finite time interval \([T_0, T]\) to the infinite time interval \((-\infty, +\infty)\).

The input-output map of (1) with \( x(-\infty) = 0 \) is denoted as follows:

\[
y(t) = \int_{-\infty}^{t} c e^{A(t-\tau)} b u(\tau) d \tau.
\]  

(23)
y is contained in \( L_1 \cap L_2 \) because \( u \) is contained in \( L_1 \cap L_2 \) and all eigenvalues of \( A \) have negative real parts. (23) is expressed by a linear operator on \( L_1 \cap L_2 \) as follows:

\[
y = Su, \quad (u, y) \in L_1 \cap L_2.
\]  

(24)

Since \( L_1 \cap L_2 \subset L_2 \), the inner product and the norm in \( L_1 \cap L_2 \) are defined as \( \langle u, y \rangle \Delta = \int_{-\infty}^{\infty} u(t)^T y(t) dt \) and \( \| u \| \Delta \sqrt{\langle u, u \rangle} \), respectively. And, \( \mathcal{R}(S) \) and \( \mathcal{N}(S) \) denote the range and the null space of an operator \( S \), respectively. Like the adjoint-type ILC on the finite time interval, \( S \) is decomposed as \( S = \hat{S} \hat{U} \). Then, the known part \( \hat{S} \) is used in update law. The adjoint operator \( \hat{S}^* \) of \( \hat{S} \) is denoted as follows:

\[
\hat{S}^* y(t) = \int_{t}^\infty \hat{b}^T e^{\hat{S}^T (t-\tau)} e^T y(\tau) d \tau.
\]  

(25)
The next theorem is shown.

**Theorem 1.** Suppose \( y_d^{(2)} \in L_1 \cap L_\infty \). If the unknown part \( U \) satisfies the following inequality

\[
\langle U \eta, \eta \rangle \geq \beta \| \eta \|^2_{2}, \quad (\beta > 0),
\]  

(26)
for all \( \eta \in L_1 \cap L_\infty \), then the sequence \( \{ u_k; k = 0, 1, \ldots \} \) generated by

\[
\begin{align*}
 u_{k+1} &= u_k - \alpha \hat{S}^* (Su_k - y_d) \\
 u_0 &= 0
\end{align*}
\]  

(27)
satisfies

\[
\left\| u_k - u_d \right\|_2 \to 0, \quad (k \to \infty)
\]  

(28)
where, \( u_d \in L_1 \cap L_\infty \) satisfies \( y_d = Su_d \) and the constant \( \alpha \) is chosen as \( 0 < \alpha < 2\beta \| \hat{S} \| \| \hat{U} \| \).  

**Proof of Theorem 1.** From \( y_d^{(2)} \in L_1 \cap L_\infty \) and the theory of stable inversion, there exists \( u_d \) such that \( y_d = Su_d \). And from (16), \( u_d^{(2)} \) is contained in \( L_1 \cap L_\infty \). Since the relative degree of the system that the input/output map is \( \hat{S}^* \) is \( r \), there exists \( v_j \in L_1 \cap L_\infty \) such that \( u_d = \hat{S}^* v_j \) as the above. That is, the following equation is satisfied

\[
\hat{S}^* v_j \in L_1 \cap L_\infty, \quad y_d = \hat{S}^* v_j.
\]  

(29)
(29) implies that there exists the unique \( v_j \in \mathcal{N}(\hat{S}^*)^* \) such that \( \hat{S}^* v_j = u_d \), where \( \mathcal{N}(\hat{S}^*)^* \) denotes the orthogonal complement of \( \mathcal{N}(\hat{S}^*) \). Furthermore, since (27) with \( u_0 = 0 \in \mathcal{R}(\hat{S}) \) implies \( u_d \in \mathcal{R}(\hat{S}) \), \( (k = 0, 1, \ldots) \), there exists the unique \( v_k \in \mathcal{N}(\hat{S}^*)^* \) such that \( \hat{S}^* v_k = u_d \), \( (k = 0, 1, \ldots) \). Therefore, (27) is denoted

\[
\hat{S}^* (u_{k+1} - v_d) = \hat{S}^* (u_k - v_d) - \alpha \hat{S} (\hat{S}^* U \hat{S}^* (u_k - v_d))
\]  

(30)
Since \( \mathcal{R}(\hat{S}) \subset \mathcal{R}(\hat{S})^* \), where \( \mathcal{R}(\hat{S})^* \) indicates the closure of \( \mathcal{R}(\hat{S}) \),

\[
v_k - v_d = \hat{S} U \hat{S}^* \hat{S} (u_k - v_d) \in \mathcal{N}(\hat{S}^*)^*
\]  

for \( k = 0, 1, \ldots \). Hence, the equation

\[
v_{k+1} - v_d = v_k - v_d - \alpha \hat{S} U \hat{S}^* (u_k - v_d)
\]  

(31)
is satisfied because \( \hat{S}^* \) is invertible on \( \mathcal{N}(\hat{S}^*)^* \). From (31) and (26), we have

\[
\left\| v_{k+1} - v_d \right\|_2 \leq \| v_k - v_d \|^2_2 - \alpha 2\beta \| \hat{S} \| \| \hat{U} \|^2_2 \left\| \hat{S}^* (u_k - v_d) \right\|_2^2
\]  

(32)
Since \( \alpha \| \hat{S} U \hat{S}^* \|^2_2 > 0 \) and \( \left\| v_k - v_d \right\|_2 ; k = 0, 1, \ldots \) are bounded from below, we establish

\[
\left\| \hat{S}^* (u_k - v_d) \right\|_2 \to 0 \quad (k \to \infty)
\]  

(33)
as \( k \to \infty \).

**Remark 3.** In [8], ILC using stable inversion was dis-
cussed for non-minimum phase systems. The update law of this method uses stable inversion directly as follows:

\[ u_{k+1} = u_k + S_{SI}^{-1}(y_k - y_d) \]  

(34)

where \( S_{SI}^{-1} \) denotes the stable inversion of the system \( S \). Thus, we can easily expect that this method yields the desired input defined by stable inversion. On the other hand, there is no direct relation between the update law using adjoint system and stable inversion. However, this update law also yields the desired input defined by stable inversion. This is the most interesting aspect of this work.

**IV. CONVERGENCE CONDITION ON THE FREQUENCY DOMAIN**

The inequality (26) expresses the convergence condition on the time domain. However, it is difficult to judge that the unknown part \( U \) satisfies this condition or not. In this section, we develop a simple convergence condition on the frequency domain.

The transfer functions of (1) and the known part \( \hat{S} \) are denoted by \( S(s) \) and \( \hat{S}(s) \), respectively, that is,

\[ S(s) = c(sI - A)^{-1}b \]  

(35)

\[ \hat{S}(s) = \hat{c}(sI - \hat{A})^{-1}\hat{b}. \]  

(36)

Hence, the transfer function of the unknown part \( U \) is

\[ U(s) = S(s)\hat{S}(s)^{-1}. \]  

(37)

In [3], a convergence condition of adjoint-type ILC on a finite time interval was discussed on the frequency domain as follows: if \( U(s) \) satisfies strictly positive realness and \( U(\omega) > 0 \), then the unknown part satisfies the inequality (10).

We can consider the similar approach which these conditions are extended to adjoint-type ILC on an infinite time interval. However, it is impossible to adapt this approach for non-minimum phase systems because \( U(s) \) never satisfies strictly positive realness due to existence of the unstable zeros. Hence, we develop a convergence condition on the frequency domain from a different approach.

**Theorem 2.** Suppose that \( y^{(\alpha)} \in L_1 \cap L_\infty \) and \( \hat{S}(s) \) has no zeros on the imaginary axis. Moreover, \( r + \tilde{r} \) is even where \( \tilde{r} \) is the relative degree of \( \hat{S}(s) \). If the unknown part \( U(s) \) satisfies the following inequality

\[ \forall \omega \in \mathbb{R} \quad \text{Re} \ U(j\omega) > 0, \]  

then the sequence \( \{u_k; k = 0, 1, \ldots\} \) generated by

\[ u_{k+1} = u_k - \alpha_2 \hat{S}^\dagger(Su_k - y_d) \]  

(39)

\[ u_0 = 0 \]

satisfies

\[ \|u_k - y_d\|_2 \to 0, \quad (k \to \infty) \]  

(40)

where \( u_d \in L_1 \cap L_\infty \) satisfies \( y_d = Su_d \) and the constant \( \alpha_2 \) is chosen as

\[ 0 < \alpha_2 < \min_{\omega \in \Omega_{\alpha}} \frac{2\text{Re} \hat{S}(j\omega)}{|\hat{S}(j\omega)|^2}. \]  

(41)

**Proof of Theorem 2.** Since \( S \) and \( \hat{S} \) have no zeros on the imaginary axis, the following relation is obtained. For all \( \omega \in \mathbb{R} \)

\[ \text{Re} U(j\omega) > 0 \iff \text{Re} \ \hat{S}(j\omega) \text{Re} S(j\omega) > 0 \]  

(42)

\( \varepsilon_k \) denotes Fourier transformation of \( \varepsilon_k \) where \( \varepsilon_k \overset{\Delta}{=} u_k - u_d. \) Since \( \varepsilon_k \in L_1 \cap L_\infty \subset L_2 \), for all \( \epsilon > 0 \) there exists \( \Omega > 0 \) such that

\[ \int_{|\omega|=1-\epsilon} \left| \varepsilon_k(j\omega) \right|^2 d\omega < \epsilon \]  

(43)

by Parseval equality.

From (38), (41) and (42), we have the following inequality

\[ \sup_{\omega \in (-\infty, +\infty)} \left| 1 - \alpha_2 \hat{S}(j\omega) \right| \leq 1 \]  

(44)

and

\[ \sup_{\omega \in (-\infty, +\infty)} \left| 1 - \alpha_2 \hat{S}(j\omega) \right| \leq \rho < 1. \]  

(45)

The remainder of this proof is shown by using the same method in [10].

Finally, if \( r + \tilde{r} \) is even, then the existence of minimum for the right hand side of (41) is guaranteed. ■

By using this theorem, we can easily judge that the input sequence converges to the desired input without influence of the unstable zeros.

**Example 2.** In this example, we show that it is possible to judge convergence to the desired input only (38). Consider the same system

\[ S(s) = \frac{s^2 - 1}{s^3 + 4s^2 + 6s + 4} \]  

(46)

and desired output as example 1. Figure 7 shows the
Nyquist diagram of \( U(j\omega) = S(j\omega)\hat{S}(j\omega)^{-1} \) when we chose \( \hat{S}(s) \) as follows:

\[
\hat{S}(s) = \frac{s - 2}{s^2 + 6s + 4} \quad (47)
\]

From this Nyquist diagram, it is possible to converge to the desired input by (39). When the trial interval is \([-2\pi, 3\pi]\), outputs and inputs of adjoint-type ILC and stable inversion are shown in Figs. 8 and 9, respectively.

**V. CONCLUSION**

ILC for non-minimum phase systems has been the focus of several papers [6-8]. In this paper, we investigate ILC for non-minimum phase systems from the viewpoint of the relationship between adjoint-type ILC and stable inversion. We showed that these methods give the equivalent desired input. We also present the convergence conditions on the time and frequency domain. We can easily judge the convergence to the desired input by using Nyquist diagram. However, the update law in this paper is applied on the infinite time interval. When the adjoint-type ILC is applied to actual non-minimum phase systems, it is impossible to carry out the trial on an infinite time interval. The trial interval is necessarily truncated to the finite interval. In [11], the effects of the truncation of the time interval on the tracking performance are investigated. If the trial interval is sufficiently large, then we can keep the good control performance.

Since ILC is suitable for the control of uncertain systems and stable inversion is suitable for control of non-minimum phase systems, the adjoint-type ILC is an effective method for non-minimum phase systems with uncertainly.

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