RESTRICTED STRUCTURE CONTROL LOOP PERFORMANCE ASSESSMENT FOR PID CONTROLLERS AND STATE-SPACE SYSTEMS

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ABSTRACT

A novel $H_2$ optimal control performance assessment and benchmarking problem is considered for discrete-time state-space multivariable systems, where the structure of the controller is assumed to be fixed apriori. The controller structure may be specified to be of PID, reduced order, or lead/lag forms. The theoretical problem considered is to represent the state-space model in discrete polynomial matrix form and to then obtain the causal, stabilising, controller, of a prespecified form, that minimises an $H_2$ criterion. This then provides the performance measure against which other controllers can be judged. The underlying practical problem of importance is to obtain a simple method of performance assessment and benchmarking low order controllers.

The main theoretical step is to derive a simpler cost-minimization problem whose solution can provide both the full order and restricted structure (PID) optimal benchmark cost values. This problem involves the introduction of spectral factor and diophantine equations and is solved via a Wiener type cost-function expansion and simplification. The numerical solution of this problem is straightforward and involves approximating the simplified integral criterion by a fixed number of frequency points. The main benchmarking theorem applies to multivariable systems that may be unstable, non-minimum phase and non-square.

KeyWords: PID control, design, tuning, optimization, process control.

I. INTRODUCTION

Controller performance benchmarking is important industrially, since there are thousands of loops in a typical process plant and many of these will not have been tuned adequately. The competitive pressures between companies make it very desirable to obtain the best from the process control system. The investment in control loop Supervisory Control and Data Acquisition (SCADA) systems is also significant and poorly tuned controllers can waste this investment.

Harris (1989 [1]) and Desborough and Harris (1992 [2], 1993 [3], 1994 [4]) considered the assessment of control loop performance for both feedback and feedforward control using minimum variance as the benchmark cost measure. Huang and Shah (1999 [5]) summarised the state of the art in a monograph, mostly focusing on the minimum variance cost index as the performance assessment measure. There has been growing interest in controller performance benchmarking over the last decade, often learning from the business process benchmarking community (Codling 1992 [6], Rolstadas 1995 [7], Anderson and Petersen 1996 [8], Ahmad and Benson 1999 [9]).

The minimum variance criterion has some value as a benchmark, since the economic performance of a process is often governed by how close a set-point can be moved to an operating boundary (such as a temperature limit). By reducing the variance of the output deviations setpoints can be moved closer to such boundaries. However, there are real problems in implementing minimum variance controllers, since they have wide

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bandwidth, noise amplification and lead to very aggressive control action. Under normal circumstances such controllers would result in saturation and excessive wear and tear on the actuators. Minimum variance does not therefore by itself provide a very practical benchmark.

The strategy followed is to minimise an $H_2$ or LQG criterion in such a way that the controller is of the desired form and is causal. A simple analytic solution cannot be obtained, as in the case where the controller structure is unconstrained (Kucera (1979 [10]). However, a relatively straightforward direct optimization problem can be established which provides the desired solution. The main contribution lies in the preprocessing, which enables a simpler cost minimisation problem to be obtained, which is then easy to solve numerically. The preprocessing distinguishes the approach from other low-order optimal controller tuning methods (River and Morari 1990 [11]; Anderson and Moore 1971 [12]; Vinnicombe and Miyamoto 1997 [13]; Takahashi et al. 1997 [14]; Hickey et. al 1999 [15]; Chen et al. 1995 [16]; Saeki et al. 1994 [17]; 1996 [18], 1998 [19], 1997 [20]; Hovd and Skogestad 1994 [21]; Iwasaki and Skelton 1995 [22]; by, Bryson and Cannon 1985 [23] and Wenk and Knapp 1980 [24]).

The main aim of a benchmarking technique is to diagnose control loop performance by providing tools to determine:

(i) The cost index that may be employed as a benchmark.
(ii) The best achievable performance under the constraints on the controller structure, in terms of the benchmark.
(iii) The shortfall in performance that the actual control loop, with possibly sub-optimal controller, provides.

The performance assessment of control loops is considered for systems represented in state equation form. The $H_2$ criterion is employed to judge performance and the same form of performance metrics may be used for condition monitoring. This enables the relatively new problem of benchmarking and condition monitoring restricted structure or PID designed control systems [11] to be explored. The system description will be introduced first.

**Roadmap for the Paper**

The system is represented in state equation form in §2 and involves separate tracking and plant subsystems. The performance criterion, which represents the benchmark cost is defined in §3, and the restricted structure controller is introduced. This section also includes a theorem which enables the benchmark cost computation to be simplified. The results are based on the solution of polynomial system equations. The actual numerical solution and computation of the benchmark cost values is the only place where numerical approximation is involved and the algorithm is presented in §4. A detailed design example is presented in §5 to illustrate the type of results obtained and to explore how they may be interpreted.

**II. SYSTEM DESCRIPTION**

The system of interest is assumed to be represented in stabilizable and detectable state equation model form. However, the restricted structure control problem is solved using a unit delay transfer function operator representation and these models are also defined. The system is shown in Fig. 1. The reference (subsystem $S_0$) and plant (subsystem $S_1$) are shown separately, so that the characteristics of the two degrees-of-freedom restricted structure solution can be investigated.

The zero-mean white disturbance and noise sources $\{\xi_0(t)\}, \{\xi_1(t)\}$ and $\{v_0(t)\}, \{v_1(t)\}$ are assumed statistically independent. The disturbance model covariance matrices may be assumed to be normalized to the identity and the measurement noise covariance matrices are denoted by $R_f$ and $R_m$, respectively. The state, observations and control law equations, may be given as:

**Reference $S_0$:**

\[
x_0(t+1) = A_0 x_0(t) + D_0 \xi(t) \\
z_0(t) = C_0 x_0(t) + v_0(t)
\]

**Plant $S_1$:**

\[
x_1(t+1) = A_1 x_1(t) + B u(t) + D_1 \xi(t) \\
z_1(t) = C_1 x_1(t) + v_1(t)
\]
Control law: 
\[ u(t) = -K_0(z^{-1})z_0(t) - K_1(z^{-1})z_1(t) \]  
(5)

2.1 Augmented system model

Combining the state equations for the reference, plant and noise model obtain the total augmented system as:

\[
\begin{bmatrix} x_0(t+1) \\ x_1(t+1) \end{bmatrix} = \begin{bmatrix} A_0 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} x_0(t) \\ x_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u(t) + \begin{bmatrix} D_0 \\ 0 \end{bmatrix} \xi_0(t) + \begin{bmatrix} 0 \\ D_1 \end{bmatrix} \xi_1(t)
\]

which may be written, with an obvious definition of terms, in the more concise form:

\[
X(t+1) = A X(t) + B u(t) + D_1 \xi(t)
\]

(6)

Similarly, the observations signal may be defined as:

\[
\begin{bmatrix} z_0(t) \\ z_1(t) \end{bmatrix} = \begin{bmatrix} C_0 \\ 0 \end{bmatrix} x_0(t) + \begin{bmatrix} 0 \\ C_1 \end{bmatrix} x_1(t) + \begin{bmatrix} v_0(t) \\ v_1(t) \end{bmatrix}
\]

which may be written as:

\[
z(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} X(t) + v(t) = C X(t) + v(t)
\]

(9)

2.2 Definition of transfer operators

The transfer operators for the system, in terms of the operator \( z^{-1} \), may be written as:

\[ \Phi(z^{-1}) = (I - A)^{-1} \]

(10)

\[ \tilde{\Phi}(z^{-1}) = \Phi(z^{-1})B, \quad W(z^{-1}) = C_2 \Phi(z^{-1})B \]

(11)

\[ \tilde{\Phi}_s(z^{-1}) = \Phi(z^{-1})D_{11}, \quad W_s(z^{-1}) = C_1 \Phi(z^{-1})D_{11} \]

(12)

\[ \tilde{\Phi}_d(z^{-1}) = \Phi(z^{-1})D_{22}, \quad W_d(z^{-1}) = C_2 \Phi(z^{-1})D_{22} \]

(13)

2.3 Polynomial systems design

The following polynomial matrices may now be introduced (with a subscript \( p \)). Let \( \tilde{A}_p = (I - z^{-1}A) \) and denote the transfer to system states as:

\[ \tilde{\mathcal{H}} = \begin{bmatrix} \tilde{W}_s & \tilde{W}_d \end{bmatrix} = \tilde{\Phi}_s^p \begin{bmatrix} z^{-1}B & z^{-1}D_{11} \end{bmatrix} \]

(14)

The right-coprime form for \( \tilde{\mathcal{H}} \) may be written as:

\[ \tilde{\mathcal{H}} = \tilde{\mathcal{T}}_s^{-1} \tilde{W}_p = \tilde{\mathcal{B}}_p \tilde{A}_p \]

(15)

where \( \tilde{W}_p = z^{-1}B \). The right-coprime form for the plant transfer may also be written as:

\[ W = C_{21} \tilde{T}_s^{-1}z^{-1}B = B_{1p} A_{1p} \]

(16)

and by comparison of (15) and (16) \( B_{1p} = \tilde{B}_p \) and \( A_{1p} = \tilde{A}_p \). The transfer from state inputs to outputs may be written as:

\[ W_l^{-1} = \begin{bmatrix} C_{11}, z^{-1} & C_{21} z^{-1} \end{bmatrix} \tilde{A}_p^{-1} \]

(17)

The left-coprime forms for \( W_l^{-1} \) and \( W_2 \) may be written as:

\[ W_l^{-1} = C_{11} z^{-1} \tilde{T}_s^{-1} = A_{1p} C_{11} \]

(18)

\[ W_2 = C_{21} z^{-1} \tilde{T}_s^{-2} = A_{21} C_{21} \]

(19)

The following left coprime forms for \( W, W_l^{-1} \) and \( W_2 \) may be written as:

\[ W = W_{1p} B = A_{1p} C_{1p} \]

(20)

\[ W_l^{-1} = W_{1p} D_{11} = A_{1p} C_{1p} \]

(21)

\[ W_2 = W_{2p} D_{22} = A_{2p} C_{2p} \]

(22)

and it also follows, by comparison of (16), (18), (19) with (20) to (22):

\[ A_p = A_{1p} \quad B_{1p} = C_{1p} B \]

(23)

III. PERFORMANCE METRIC AND MINIMIZATION PROBLEM

The cost minimization problem involves penalising states, rather than outputs, which is valuable, since by appropriate definition of state weighting \( \tilde{Q} = H^T H \) the set of inferred outputs: \( y_{\text{hf}}(t) = H x(t) \) may be costed. The main performance benchmarking index to be introduced is based on the following \( H_2 \) criterion:

\[
J = \lim_{t \to \infty} \left[ E \left\{ \frac{1}{T} \sum_{t=0}^{T} \| X^T(t) \tilde{Q} X(t) + X^T(t) \tilde{Q} X(t) + u^T(t) \tilde{Q} u(t) + u^T(t) \tilde{Q} u(t) \| \right\} \right]
\]
This may be written, using Parseval’s theorem, as the
sum of three terms: \( J = J_x + J_{xu} + J_u \). That is
\[
J = \frac{1}{2\pi} \text{trace}\{[\mathcal{H}_x^T \Phi_x(z^{-1}) + 2\mathcal{H}_x \Phi_{xx}(z^{-1})] \}
+ \text{trace}\{[\mathcal{H}_u \Phi_{uu}(z^{-1})] \}
\]
where the constant weighting matrices satisfy \( \mathcal{Q}_c \geq 0 \) and \( \mathcal{R}_c \geq 0 \).

The unconditional expectation \( \mathbb{E} \{.\} \) is taken over
the ensemble \( \{\xi(t)\} \) and \( \{\nu(t)\} \). The problem is to mini-
mise the above criterion and obtain the benchmark cost
values for a controller restricted to have a specified
structure. Some typical controller structures, for the sca-
lar case, include:

**Unrestricted structure but reduced order:**
\[
C(z^{-1}) = \frac{c_{p0} + c_{p1}z^{-1} + \cdots + c_{pn}z^{-n}}{c_{z0} + c_{z1}z^{-1} + \cdots + c_{zr}z^{-r}}
\]
where \( r \geq p \) is less than the order of the system (plus
weightings).

**Lead lag compensator:**
\[
C(z^{-1}) = \frac{c_{p0} + \alpha c_{p1}z^{-1}}{c_{z0} + \alpha c_{z1}z^{-1}}
\]

**Traditional PID:**
\[
C(z^{-1}) = k_3 + k_2/(1-z^{-1}) + k_1 z^{-1}
\]

**Filtered PID:**
\[
C(z^{-1}) = k_4 + k_1/(1-z^{-1}) + k_2 z^{-1} + k_3 (1-\alpha z^{-1})
\]

where in state equation form \( (k_1 = k_0 + k_1 + k_2) \):
\[
\begin{bmatrix}
x_{1}(t+1) \\
x_{2}(t+1)
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\alpha & 1
\end{bmatrix} \begin{bmatrix}
x_{1}(t) \\
x_{2}(t)
\end{bmatrix} + \begin{bmatrix}
1 \\
\alpha - 1
\end{bmatrix} e(t)
\]
\[
u(t) = \begin{bmatrix}
k_1 \\
k_2
\end{bmatrix} \begin{bmatrix}
x_{1}(t) \\
x_{2}(t)
\end{bmatrix} + k_3 e(t)
\]

The benchmarking performance optimization must be
performed with respect to the parameters which are not
fixed, such as the three gains \( k_1, k_2 \) and \( k_3 \) in the fil-
tered PID case. The assumption must of course be made
that a stabilising control law exists for the assumed con-
troller structure and the optimal control solution is re-
quired to be causal.

Consider first the results for the unrestricted struc-
ture full-order optimal solution, since some of the dio-
phantine and spectral-factor equations are the same for
the restricted and full-order control cases.

**Theorem 3.1. State Costed Output Feedback Full-Order
H_2 Optimal Controller**
The optimal output feedback, two degrees-of- freedom,
controller for the polynomial model of the state-space
system, described in §2, to minimise the \( H_2 \) criterion (24),
may be found from the following diophantine and spec-
tral factor equations.

**Control and filter spectral factors:**
\[
D_p D_p = \mathbf{B}_p \mathbf{Q}_p \mathbf{B}_p + \mathbf{A}_p \mathbf{Q}_p \mathbf{A}_p + \mathbf{B}_p \mathbf{G}_p \mathbf{A}_p
+ \mathbf{A}_p \mathbf{G}_p \mathbf{B}_p
\]
\[
D_{dp} D_{dp} = C_{dp} C_{dp} + A_{dp} R_{dp} A_{dp}
\]
\[
D_{dp} D_{dp} = C_{dp} C_{dp} + A_{dp} R_{dp} A_{dp}
\]

**Regulating loop diophantine equations:**
\[
z^{-\tau_1} D_{p} G_{p} + F_{p} A_{p} = (B_{p} \mathbf{Q}_p + \mathbf{A}_p \mathbf{G}_p \mathbf{B}_p) z^{-\tau_1}
\]
\[
z^{-\tau_2} D_{p} H_{p} - F_{p} B_{p} = (\mathbf{A}_p \mathbf{G}_p \mathbf{B}_p) z^{-\tau_2}
\]

**Filtering diophantine equations:**
\[
\begin{align*}
G_{p} & = \frac{c_{p0} + c_{p1}z^{-1}}{c_{z0} + c_{z1}z^{-1}} \\
F_{p} & = \frac{c_{d0} + c_{d1}z^{-1}}{c_{z0} + c_{z1}z^{-1}}
\end{align*}
\]

The full-order optimal control tracking and feed-
back components may be computed using:
\[
u(t) = -K_3 (z^{-1}) \xi(t) - K_1 (z^{-1}) \nu(t) = -K (z^{-1}) \nu(t)
\]
where the dynamic gains:
\[
K(z^{-1}) = (I + K_2 (\mathbf{A}_p + K_1 C_{dp})^{-1} K_1 \mathbf{A}_p)
+ K_3 (C_{dp})^{-1} K_1 \mathbf{A}_p + K_3 (C_{dp})^{-1} K_1
\]
and the constant matrix gains:
\[ K_r = (H_{sp})^{-1} G_{sp}, \quad K_{ro} = G_{sp}(H_{sp})^{-1}, \]
\[ K_{f} = G_{sp}(H_{sp})^{-1} \]

**Stability:**

The degree of stability of the closed-loop, full-order, unrestricted structure control system, depends upon the roots of the following implied equations:

Control:

\[ G_{sp} B_0 + H_{sp} \bar{A}_0 = D_{sp} \]  
(36)

Disturbance filter:

\[ \bar{C}_{sp} G_{sp} + A_p H_{sp} = D_{sp} \]  
(37)

Reference filter:

\[ \bar{C}_{sp} G_{sp} + A_p H_{sp} = D_{sp} \]  
(38)

**Optimum full-order controller benchmark cost:**

\[ J_{\text{m}} = \frac{1}{2\pi} \left( \text{trace}(T_2 T_2^*) + \text{trace}(T_3 T_3^*) + I_{ao} \right) \frac{dz}{z} \]  
(39)

where the integrand terms:

\[ T_2 = (G_{sp} z^{-1} F_{s} z^{5} + D_{sp} z^{-1} D_{c} C_{sp} z^{0} D_{sp} D_{sp} z^{-1}) D_{sp} z^{-1} \]  
(40)

\[ I_{ao} = \text{trace}(\bar{W}_{s} \bar{W}_{s}) \]
\[ - \text{trace}(\bar{Y}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s} \bar{Y}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s}^{-1}) \]
\[ + \text{trace}(\bar{W}_{s} \bar{W}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s} \bar{Y}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s}^{-1}) \]  
(41)

and \( \bar{W}_{s} + \bar{W}_{s} \bar{W}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s} \bar{Y}_{s} \bar{W}_{s} \bar{W}_{s} \bar{Y}_{s}^{-1} \)

**Proof.** The proof of the above full-order results are also provided in the first part of Appendix 1 and can be compared with the output costed solutions in Grimble (2001) [25].

3.1 Restricted structure control and performance benchmark

It is likely that an existing process plant will have a classically designed controller, of more conventional structure (like a PID controller), than it is to use a full-order LQG solution. The value of the full order cost is given by Theorem 3.1, equation (39). Although it is of some value to compare this cost \( J \) with the best that can be achieved \( J_{\text{m}} \), this is often an unfair benchmark comparison. The reason is of course that the restriction on the controller structure and the order, may make the absolute minimum cost \( J_{\text{m}} \) way below what is achievable using the best tuned classical controller.

A more appropriate benchmark figure is therefore to compare the actual cost with that of the optimal cost assuming the controller structure is fixed. Many authors have considered this type of constrained structure optimal control problem (see [11] to [20]) but not in terms of the performance assessment and benchmarking problem.

**Theorem 3.2. Restricted Structure Optimal Control Benchmark**

Given the solution of the spectral factorisation (25) to (27) and the diophantine equations (28) to (33), the optimal controller of restricted structure, to minimise the cost function (24), can be found by minimising the simplified feedback and reference tracking criterion:

\[ J_b = J_r + J_s = \frac{1}{2\pi} \left( \text{trace}(T_2 T_2^*) + \text{trace}(T_3 T_3^*) \right) \frac{dz}{z} \]  
(42)

where

\[ T_2 = H_{sp}(I + K_{s} B_{s} K_{s}^{-1}) \]  
(43)

and

\[ T_3 = D_{sp}(A_{sp} + K_{s} B_{sp})^{-1} K_{s}(z^{-1}) A_{sp}^{-1} D_{sp} \]  
(44)

and the sensitivity function:

\[ S_{rf} = (z \bar{A}_{f} + K_{s} C_{sp})^{-1} \]  
(45)

If the controller has a restricted or limited structure, the minimum of the cost term \( J_{\text{m}} \) will be non-zero. The minimum value of the criterion (24) will then be obtained as \( J = J_{\text{m}} + J_{\text{m}} \). For an unconstrained solution the minimum is achieved when \( T_2^* = 0 \) and \( T_3^* = 0 \) and the minimum of \( J_b \) (denoted \( J_{\text{m}} \)) is zero.

**Proof.** The proof follows directly from the results in Appendix 1.

**Remarks**

(i) The above results are true for any controller and are therefore valid for the cases when either the feed-
back loop is optimal \((T^*_b = 0)\) or the tracking control is optimal \((T^*_c = 0)\).

(ii) The equation (42) reveals the increase in cost which occurs by restricting the controller structure. That is,
\[
\Delta J = J^*_c + J^*_b = \frac{1}{2\pi} \text{trace}(T^* T^{*\prime}) \int_0^\pi \frac{dz}{z}
\]
\[
+ \frac{1}{2\pi} \text{trace}(T^* T^{*\prime}) \int_0^\pi \frac{dz}{z}
\]

(iii) The Controller Performance Index will be defined as the ratio of the minimum possible variance of the cost function (24), to the actual variance:
\[
\kappa = \frac{\min_{J_d}}{\min_{J_r}} = 1 - J_d(J_{na} + J_d)
\]

and the CPI clearly lies between 0 \(\leq \kappa \leq 1\). If \(\kappa\) is close to unity the system provides little opportunity for improvement. If the CPI is close to zero retuning is recommended. This scalar is similar but not the same, as the assessment measure introduced by Desborough and Harris (1992 [2]).

IV. PARAMETRIC OPTIMIZATION PROBLEM

There are many numerical optimization algorithms that can be applied to the computation of the benchmark cost values for a particular non-optimal (low order) controller. A successive approximation algorithm will be introduced below that has proved very successful in applications. For simplicity the single-input single-output case will be considered in this section, since the extension to the multivariable problem is straightforward.

It has been shown in Appendix 1 that the computation of the optimal feedback controller \(K(z^{-1})\) reduces to minimization of the term \(J_{d}\):
\[
J_{d} = \frac{1}{2\pi} \text{trace}(T^* T^{*\prime}) \int_0^\pi \frac{dz}{z}
\]
\[
= \frac{T}{2\pi} \int_0^{\pi/2} \{T^* (e^{-i\theta}) T^{*\prime} (e^{i\theta})\} d\theta
\]
and from (44) the tracking transfer can be written as:
\[
T^*_c = (C_{s0} L_c - C_{s0} L_d)
\]
where the tracking controller: \(K_{s} = C_{s0} C_{s1}^{-1}\) also has a restricted prespecified structure.

The numerical algorithm to evaluate the feedback cost (46) and tracking cost (47) involves approximating these integrals by a summation. The resulting cost is parameterized in terms of the unknown restricted structure controller coefficients, and the optimization is then straightforward. This is described in Grimble (2000 [28]) and the main steps are summarized in Appendix 2.

V. RESTRICTED STRUCTURE PID CONTROL PERFORMANCE ASSESSMENT EXAMPLE

The following example illustrates the computation of a restricted structure controller and the way in which the benchmark performance measures may be used to assess quality of control. The system is represented in state equation form and it includes various subsystems. The first subsystem \(S_0\) represents a reference model. The second subsystem \(S_1\) represents the plant and disturbance models and the third subsystem \(S_2\) is the representation of a colored measurement noise model. The fourth subsystem \(S_3\) represents a dynamic weighting on control action.

The reference model represents an approximate integrator driven by white noise, which is the stochastic equivalent of a step function model. The plant model represents a time constant and the same plant subsystem has an input from a disturbance model, which also represents an approximate integrator. The colored measurement noise model is a lightly damped second order system and there is a lead term representing a control signal cost weighting function.

Continuous-Time Models

Reference model \(S_0\):
\[
W_s = \frac{200}{10000s + 1}
\]

Disturbance model \(S_1\):
\[
W_s = \frac{10000}{(s + 1)(10000s + 1)}
\]

Noise model \(S_2\):
\[ W_e = \frac{2.5s}{s^2 + 2s + 0.25} \]  

(50)

Control weighting model \( S_2 \):

\[ H_s = \frac{0.01(20s + 1)}{s + 1} \]  

(51)

5.1 State equation matrices

After discretization the augmented system model \( S \) has the form:

\[
\begin{pmatrix}
0.9999 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.3678794 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.9999 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.8187308 \\
0 & 0 & 0 & 0 & 0.3678794 & 1.5968225 \\
0 & 0 & 0 & 0 & 0 & 0.3678794
\end{pmatrix}
\]

\[ A = \begin{pmatrix}
0 \\
0.7950999 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \]

\[ B = \begin{pmatrix}
10 & 0 & 0 & 0 & 0 \\
0 & 0.7950999 & 1.0000250 & 2.1726894 & 2.1726894 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0144247
\end{pmatrix}
\]

\[ C = \begin{pmatrix}
0.6321838 \\
0.1849143 \\
-0.6321364 \\
0 \\
-0.0240206
\end{pmatrix}
\]

\[ D_{diag} = \begin{pmatrix}
0.0199990, 0.7950999, 1.0000250, 0, 1, 0
\end{pmatrix} \]

5.2 Cost-function weighting definitions

The following cost-function weightings were defined for the physical problem of interest and have the form:

\[
\begin{pmatrix}
0.9999 \\
0 & 0.3678794 \\
0 & 0 & 0.9999 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.8187308 \\
0 & 0 & 0 & 0 & 0.3678794
\end{pmatrix}
\]

\[ A = \begin{pmatrix}
0 \\
0.7950999 \\
0 \\
0 \\
0 \\
1
\end{pmatrix} \]

\[ B = \begin{pmatrix}
10 & 0 & 0 & 0 & 0 \\
0 & 0.7950999 & 1.0000250 & 2.1726894 & 2.1726894 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0144247
\end{pmatrix}
\]

\[ C = \begin{pmatrix}
0.6321838 \\
0.1849143 \\
-0.6321364 \\
0 \\
-0.0240206
\end{pmatrix}
\]

\[ D_{diag} = \begin{pmatrix}
0.0199990, 0.7950999, 1.0000250, 0, 1, 0
\end{pmatrix} \]

5.3 Parameterization of the restricted structure feedback controller

The restricted structure controller is parameterized as a 3 term device (near PID) with coefficient polynomials:

\[
\alpha_0(z^{-1}) = (z-0.4)(z-0.9999)z^{-1},
\alpha_1(z^{-1}) = (z-0.4)z^{-2}, \alpha_2(z^{-1}) = (z-0.9999)^2z^{-3}
\]

and the denominator of the feedback compensator term:

\[ K_{fd}(z^{-1}) = (1-0.4z^{-1})(1-0.9999z^{-2}). \]

5.4 Parameterization of the tracking controller

\[
\alpha_0(z^{-1}) = z^{-1}, \alpha_1(z^{-1}) = (1-0.9999z^{-1})z^{-1},
\alpha_2(z^{-1}) = (1-0.9999z^{-1})^2z^{-3}
\]

and denominator of tracking compensator term:

\[ K_{td}(z^{-1}) = (1-0.8z^{-1})(1-0.9999z^{-2}) \]

5.5 Computed Feedback and Tracking Controllers

The classical (restricted structure) controllers which are to be used for the benchmarking comparison are to be tuned optimally. The computation of the best feedback controller gains is illustrated in Table 1. The computed feedback and tracking controllers were obtained as:

<table>
<thead>
<tr>
<th>Full-Order Feedback Controller:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ K_1(z^{-1}) = \frac{-0.498776(z-0.3678795)(z-0.376391)}{(z-0.368964)(z+0.5281564)} \times \frac{[(z-0.7984113)^2 + 0.4257585]}{[(z-0.633226)^2 + 0.3139305]}(z-0.9979386) ]</td>
</tr>
</tbody>
</table>

Restricted Structure Feedback Controller:
Table 2. Computed benchmark cost values.

<table>
<thead>
<tr>
<th>Cost Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>J_x</td>
<td>2.8379</td>
</tr>
<tr>
<td>J_y</td>
<td>5.8233988x10^-4</td>
</tr>
<tr>
<td>J_ε</td>
<td>2.748547</td>
</tr>
<tr>
<td>J_σ</td>
<td>1.2779x10^-3</td>
</tr>
<tr>
<td>J_min</td>
<td>5001.675</td>
</tr>
<tr>
<td>J_max</td>
<td>5004.513</td>
</tr>
<tr>
<td>J_θ</td>
<td>2.749675</td>
</tr>
<tr>
<td>κ</td>
<td>0.99945</td>
</tr>
</tbody>
</table>

Fig. 2. Bode frequency response of full order and restricted structure feedback and tracking controllers.

\[
K_c(z^{-1}) = \frac{-0.5070[(z-0.7571319)^2 + 0.4231856^2]}{z(z-0.4)(z-0.9999)}
\]

Full-Order Tracking Controller:

\[
K_t(z^{-1}) = \frac{0.6621942(z-1.273216x10^{-4})(z-0.3678795)}{(z-0.368964)(z+0.5281564)(z-0.536675)}
\times \frac{(z-0.4039789)((z-0.8068992)^2 + 0.2092932^2)}{[(z-0.6333226)^2 + 0.313905^2](z-0.9979386)}
\]

Restricted Structure Tracking Controller:

\[
K_{tR}(z^{-1}) = \frac{0.796672[(z-0.8245121)^2 + 0.1423226^2]}{z(z-0.8)(z-0.9999)}
\]

The benchmark figures of merit, that compare the full-order and restricted structure results, are shown in Table 2.

5.5 Discussion of results for the first case

The Fig. 2 shows the computed controller frequency response for the full order and reduced order cases. The feedback and tracking controller frequency responses may be compared for both full order and restricted structure designs. The controller frequency responses are reasonably similar and this is also implied in Fig. 3, which indicates the resulting sensitivity function and open-loop transfer function frequency responses, again for both the full order and the restricted structure designs.

Note that the response of the feedback loop alone is the response of a so-called one Degree of Freedom (DOF) design and this will normally contain more overshoot and be slower than the two DOF solution. The time responses, shown in Fig. 4, are remarkably similar for the full and restricted structure cases. In fact both the feedback loop response (without the action of the tracking controller) and the full 2 DOF response is almost the same for both the full order and restricted structure solutions. It is not surprising that the computed benchmark cost values, shown in Table 2, support this contention. In fact the computed value of \(κ\) is almost unity.
5.6 Modified disturbance and reference models for the second case

The above case corresponds with near integral action on both the reference and disturbance models. The time constant used was 10,000 seconds and if this is reduced to 20 seconds substantially different results occur. This is illustrated in Fig. 5 which shows the unit step response for the restricted structure and full order designs for either the feedback loop alone, (1 DOF) or the full 2 DOF tracking control.

The gains in the feedback loop are lower in this case because the disturbance model has less power at low frequency. There is therefore a steady-state error in the tracking response of the feedback loop with the full-order optimal controller. This error is reduced for the two degree of freedom design, where again almost ideal critically damped responses are obtained. This is a case where the restricted structure feedback loop has apparently a better steady-state response than the full-order optimal solution. However, the plant does not include an integrator. It follows that in the steady-state the optimal error and control will be non-zero and the DC levels will depend upon the relative size of the error and control weightings. The restricted structure steady-state response is not therefore an improvement in terms of the benchmark cost and is due to the near integral action forced into the low-order controller.

In this case there is therefore a clear disparity between the restricted structure and full order feedback control solutions, partly because of poor choice of restricted structure controller parameterization. It was in fact fixed at the same values as in the previous case. The computed benchmark cost values are shown in Table 3. It is not surprising that the computed value of \( \kappa \), which depends upon the 2 DOF tracking response and control action, is a lower in this case.

### Table 3. Computed benchmark cost values case 2.

<table>
<thead>
<tr>
<th>J_e</th>
<th>J_r</th>
<th>J_c</th>
<th>J_min</th>
<th>( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.52805</td>
<td>40.9457 \times 10^{-4}</td>
<td>0.8215907</td>
<td>1.658 \times 10^{-7}</td>
<td>4.616562</td>
</tr>
<tr>
<td>0.882065</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.7 Find remarks on the example

The example illustrated that the benchmark cost figures are a useful guide to determining the quality of control. These figures related well to the intuitive assessment based on the system time and frequency responses. The particular comparison was against optimally tuned low order (restricted structure) designs and the example also indicated the surprisingly good responses that can be achieved.

5.8 Computed controllers for second case

If the disturbance model time constant is reduced and the underlying continuous system model \( W_d = 10/(s + 1)(10s + 1) \) the computed controller gains become:

**Full-Order Feedback Controller:**

\[
K_f(z^{-1}) = \frac{-0.307739(z - 0.3678795)}{(z - 0.369536)(z + 0.4417695)} \\
\times \frac{(z - 0.3823622)(z - 0.7984113)^2 + 0.4257585^2}{(z - 0.6224563)^2 + 0.2671039^2(z - 0.9033754)}
\]

**Restricted Structure Feedback Controller:**

\[
K_f(z^{-1}) = \frac{-0.3315466(z - 0.7718527)^2 + 0.445161^2}{z(z - 0.4)(z - 0.9048374)}
\]

**Full-Order Tracking Controller:**

\[
K_t(z^{-1}) = \frac{0.4836239(z - 1.198447 \times 10^{-4})(z - 0.3678795)}{(z - 0.369536)(z + 0.4417695)(z - 0.5325366)} \\
\times \frac{(z - 0.4098772)(z - 0.7964053)^2 + 0.2169716^2}{(z - 0.6224563)^2 + 0.2671039^2(z - 0.9033754)}
\]

**Restricted Structure Tracking Controller:**

\[
K_t(z^{-1}) = \frac{0.5624595(z - 0.7861738)^2 + 0.133429^2}{z(z - 0.8)(z - 0.9048374)}
\]
VI. CONDITION MONITORING

The performance metrics have been used to judge performance and to guide controller retuning, but they may also be employed for condition monitoring purposes. There is the possibility of using performance metrics for fault monitoring which are different to those used for control design. The choice of dynamic weighting terms will be critical and those which are related to good control loop performance may be very different to those which are very sensitive to actuator/sensor degradation and faults.

A theoretical problem to consider in further research is the optimum choice of performance metrics which maximize the sensitivity to certain fault conditions. In this case the controller can be assumed to be known and fixed and the problem is to determine the best weighting function frequency responses in the condition monitoring benchmarks. The unknown coefficients in the weightings may be optimized using similar techniques to those employed in optimizing the restricted structure controller gains.

VII. CONCLUDING REMARKS

The minimum variance criterion has proved successfully in commercial industrial control systems, since the results are easy to use for controller performance assessment and benchmarking. However, there is a real difficulty in using only error variance as a benchmark cost. That is, the resulting control signal variations are normally totally unrealistic and cannot be used in practice.

A polynomial solution of the restricted structure $H_2$ optimal control problem was presented for systems represented in state-equation model form. The aim was to generate performance measures, to benchmark process control systems, that might have low-order controllers of PID structure. Such benchmark figures are more realistic then traditional minimum variance criteria, since they take into account the limitations of the system. That is, a minimum variance controller is of high order (the same as the system) and also has high gain at high frequencies, neither of which is permissible in most real processes.

The theoretical analysis was conducted and numerical algorithms developed in the frequency-domain but the underlying system was described by state equations. In many applications a physical state-space description is very suitable and yet the frequency domain provided useful insights into robust, disturbance reduction and stability properties. The approach therefore combines some of the best features of the two modeling philosophies.

The condition monitoring ideas are also novel. The performance metrics can certainly be used for loop tuning but they may have even more potential as a condition monitoring tool. Initial results suggests that the performance metrics are sensitive to possible sensor or actuator degradation/faults but this requires further investigation.

APPENDIX 1

Polynomial Matrix $H_2$ Optimal Control Solution

The system equations in §2 may be expanded to obtain the closed loop response equations:

\[
X(t+1) = AX(t) + Bu(t) + DL(t)
\]

\[
X(t) = \Phi(z^{-1})Bu(t) + \Phi(z^{-1})DL(t)
\]

The optimal control may be written in the two degrees of freedom form:

\[
u(t) = -K_s(z^{-1})z_1(t) - K_r(z^{-1})z_2(t)
\]

where

\[
z_2(t) = C_{11}X(t) + v_1(t)
\]

and

\[
z_1(t) = C_{21}X(t) + v_1(t)
\]

Note that $C_{11}\Phi B = 0$ and hence $C_{11}X(t) = C_{11}\Phi D_1 \xi(t)$

\[
u(t) = -K_s \Phi D_1 \xi(t) - K_r (Wu(t) + C_2 \Phi D_1 \xi(t))
\]

\[
- K_r v_1(t) - K_r v_1(t)
\]

or

\[
u(t) = SWV(t) - K_r (W(t) + C_2 \Phi D_1 \xi(t))
\]

where

\[
C_{11}X(t) = C_{11}X(t) + v_1(t)
\]

A1.1 Power spectral terms

The power spectra terms may be defined as follows:

\[
\Phi_{xx} = (\overline{S} \Phi S_{C_{11}} I) \overline{W} \overline{C}^* (C^* S^* \overline{W}^* - I)
\]
\[ + ( \bar{W} M C \bar{y} - I ) \bar{W} \bar{y} ( C \bar{y} \bar{y} - I ) \]
\[ + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]
\[ \Phi_{\infty} = SK \bar{W} \bar{W} + SK \bar{R} \bar{R} \bar{K} \bar{S} + SK \bar{W} \bar{W} \bar{K} \bar{S} + SK \bar{R} \bar{R} \bar{K} \bar{S} \]
\[ + \bar{W} M C \bar{y} \bar{W} \bar{y} ( C \bar{y} \bar{y} - I ) \]
\[ + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]
\[ \Phi_{a} = SK \bar{W} \bar{W} ( C \bar{y} \bar{y} - I ) \]
\[ + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]

Simplifying obtain:
\[ \Phi_{\infty} = \bar{W} S K \bar{C} \bar{W} \bar{W} + \bar{W} S K \bar{K} \bar{S} + \bar{W} M C \bar{W} \bar{W} \bar{W} + \bar{W} M R \bar{M} \bar{W} \bar{W} \]
\[ - \bar{W} W \bar{W} + \bar{W} S K \bar{C} \bar{W} \bar{W} \bar{W} + \bar{W} M C \bar{W} \bar{W} \bar{W} + \bar{W} M R \bar{M} \bar{W} \bar{W} \]
\[ + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]

The cost terms within the summation become:
\[ I_{c}(z^{-1}) = \text{trace}(\bar{G} \Phi_{\infty} - \bar{G} \Phi_{\infty}) + \text{trace}(\bar{G} \Phi_{\infty}) \]
\[ = \text{trace}(\bar{W} \bar{W} \bar{W} + \bar{W} S K \bar{C} \bar{W} \bar{W} + \bar{W} S K \bar{K} \bar{S} + \bar{W} M C \bar{W} \bar{W} \bar{W} + \bar{W} M R \bar{M} \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y}) \]
\[ + \text{trace}(\bar{W} \bar{W} \bar{W} + \bar{W} S K \bar{C} \bar{W} \bar{W} \bar{W} + \bar{W} M C \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y}) \]

\[ \text{Spectral Factors} \]

To simplify this cost expression the spectral factors : \( Y_{a}, Y_{s}, Y_{d} \) may be defined using:

\[ Y_{a} Y_{s} = \bar{W} \bar{W} + \bar{W} S K \bar{C} \bar{W} \bar{W} + \bar{W} M \bar{M} \bar{W} \bar{W} \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]
\[ Y_{a} Y_{d} = \bar{W} \bar{W} + \bar{W} S K \bar{C} \bar{W} \bar{W} + \bar{W} M \bar{M} \bar{W} \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]

A.1.2. Completing the squares

Following a conventional completing the squares argument (Kucera, 1979 [10], Grimble 2001 [25]) the cost terms may be written as follows:

\[ I_{c} = \text{trace}(Y_{a} Y_{s} Y_{d} - MW \bar{W} + \bar{W} S K \bar{C} \bar{W} + \bar{W} S K \bar{K} \bar{S} + \bar{W} M \bar{M} \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y}) \]
\[ + \text{trace}(SK \bar{W} \bar{K} \bar{S} + SK \bar{K} \bar{S} + SK \bar{W} \bar{K} \bar{S} + SK \bar{R} \bar{R} \bar{K} \bar{S}) \]

where the following term is independent of the choice of control law:

\[ Y_{a} Y_{s} = \bar{W} \bar{W} + \bar{W} S K \bar{C} \bar{W} + \bar{W} M \bar{M} \bar{W} \bar{W} + \bar{W} S R R K \bar{y} + \bar{W} M R M \bar{y} \]
The regulating cost term becomes:

\[ Y_{r}^{-1} \bar{Q} \bar{M} W Y_{r}^{-1} = (G_{i} \bar{X}_{i} + z^{-\tau} D_{i} D_{i}^{-1}) z^{-\tau} D_{i} C_{i} D_{i}^{-1} + z^{-\tau} D_{i} F_{i} z^{-\tau} D_{i} C_{i} D_{i}^{-1} \]  

(78)

To further simplify the first term in this expression note that \( C_{i} = \bar{C}_{i} D_{i} \) and introduce the filtering diophantine equation:

\[ z^{-\tau} G_{i} D_{i} \bar{Y}_{i} + \bar{A}_{i} F_{i} = D_{i} C_{i} z^{-\tau} \]  

(79)

Part of the cost term may now be written as:

\[ Y_{r}^{-1} \bar{Q} \bar{M} W Y_{r}^{-1} = G_{i} A_{i}^{-1} z^{-\tau} G_{i} F_{i} + (G_{i} z^{-\tau} F_{i} D_{i}^{-1} z^{-\tau}) + D_{i} F_{i} z^{-\tau} D_{i} C_{i} D_{i}^{-1} \]  

(80)

Regulating diophantine equations and implied equation

A further diophantine equation may be introduced to compliment the regulating equations (77).

\[ z^{-\tau} D_{i} G_{i} + F_{i} \bar{A}_{i} = (\bar{B}_{i} \bar{Q}_{i} + \bar{X}_{i} G_{i} \bar{G}_{i}) z^{-\tau} \]  

(81)

\[ z^{-\tau} D_{i} H_{i} = F_{i} \bar{B}_{i} = (\bar{A}_{i} \bar{Q}_{i} + \bar{X}_{i} G_{i} \bar{G}_{i}) z^{-\tau} \]  

(82)

Right multiplying (81) by \( \bar{A}_{i} \) and (82) by \( \bar{A}_{i} \) and adding, after division by \( D_{i} \), obtain the control implied equation:

\[ G_{i} \bar{B}_{i} + H_{i} \bar{A}_{i} = D_{i} \]  

(83)

Filtering diophantine equations and implied equation

The filtering diophantine equations, corresponding to the system model, may be written similarly as:

\[ z^{-\tau} G_{i} D_{i} \bar{Y}_{i} + \bar{A}_{i} F_{i} = D_{i} C_{i} z^{-\tau} \]  

(84)

\[ z^{-\tau} H_{i} \bar{Y}_{i} = C_{i} z^{-\tau} F_{i} = R_{i} A_{i} z^{-\tau} \]  

(85)

Note that the definition of the diophantine equations (84) and (85) depend upon whether the \( z^{-1} \) term in (77) is included or not. By not including this term the controller will have a single step delay, which is equivalent in state-space terms to using a Kalman single-step predictor, rather than Kalman filter (Grimble 2001 [25]) for state estimation. Left multiplying equation (84) by \( \bar{C}_{i} \), and (85) by \( A_{i} \) and adding obtain the first filtering implied equation:

\[ G_{i} \bar{B}_{i} + H_{i} \bar{A}_{i} = D_{i} \]  

(83)
\[ C_y G'_y + A_y H'_y = D_y \]  

(86)

Similarly, consider the tracking filter diophantine equations:
\[ z^{-T_1} G'_y D'_y + \bar{A}_p F'_p = D_1 C'_y z^{-T_1} \]  

(87)
\[ z^{-T_1} H'_y D'_y - C_y z^{-T_1} F'_p = R_0 A'_y z^{-T_1} \]  

(88)

Left multiplying (87) by \( \bar{C}_y \) and (88) by \( A_y \) and adding obtain the second filtering implied equation as:
\[ \bar{C}_y G'_y + A_y H'_y = D_y \]  

(89)

A1.5 Regulating Cost Terms \( T'_y \) and \( T'_s \)

The regulating cost term may therefore be denoted as:
\[ T_y = Y M Y_y - Y^{'T}_1 \overline{Q}_y W_y W'_y Y^{T}_1 \]
\[ = [D_y \bar{A}_p M A'_y D^{T}_y - G_y A'_p \bar{A}_p z^{-T} G'_y] \]
\[ - D^{T}_y (D G_z z^{-T} F'_p z^{-T} + F'_p z^{-T} D_2 C'_y z^{-T}) D_y^{T}_y \]  

(90)

where \( T_y = T'_y - T'_s \) and the stable \( T'_y = [\] and unstable \( T'_s = D_y^{T}_y (D_y^{T}_y) \) terms may easily be identified from this expression. If \( M = K_y (I + W K_y)^{-1} \) then \( M A_y^{T} = K_y (A_y + B_y K_y)^{-1} \). The square bracketed cost term in (90) may now be expressed as:
\[ T'_y = [D_y \bar{A}_p M A'_y D^{T}_y - G_y A'_p \bar{A}_p z^{-T} G'_y] \]
\[ = [D_y \bar{A}_p K_y - G_y A'_p z^{-T} G'_y D^{T}_y (A_y + B_y K_y)] \]
\[ (A_y + B_y K_y)^{-1} D_y \]  

(91)

The first two terms in this expression (noting \( A_y = \bar{A}_y \) and \( B_y = C_y B_y \)) become:
\[ D_y \bar{A}_p \bar{A}_p - G_y A'_p \bar{A}_p z^{-T} G'_y D^{T}_y B_y \]
\[ = G_y A'_y (B_y A_y z^{-T} G'_y D^{T}_y B_y) \]
\[ + H'_y \]  

(92)
\[ G_y A'_y (I - G'_y D^{T}_y A_y C_y z^{-T} \bar{A}_y) \bar{A}_y + H'_y \]  

(93)

Using the diophantine equation (86) the bracketed term in this last expression may be written as:
\[ (I - G'_y D^{T}_y A_y C_y z^{-T} \bar{A}_y) \]
\[ = (I - G'_y \bar{C}_y G'_y + A_y H'_y)^{-1} C_y \]
\[ = (I - G'_y \bar{H}_y^{-1} A_y C_y G'_y + I)^{-1} \bar{H}_y^{-1} A_y C_y G'_y \]
\[ = (I + G'_y \bar{H}_y^{-1} A_y C_y G'_y)^{-1} \]  

(94)

Thence, substituting in (92) using (93) obtain:
\[ D_y \bar{A}_p \bar{A}_p - G_y A'_p \bar{A}_p z^{-T} G'_y D^{T}_y B_y \]
\[ = G_y A'_y (I + G'_y \bar{H}_y^{-1} A_y C_y G'_y)^{-1} \bar{A}_y + H'_y \]  

(95)

Also
\[ D_y^{T}_y A_y = (A_y C_y G'_y + H'_y)^{-1} \]
\[ = H'_y (I + A_y C_y G'_y H'_y)^{-1} \]  

(96)

Substituting in (91) using, (94) and (95), obtain:
\[ T'_y = [D_y \bar{A}_p M A'_y D^{T}_y - G_y A'_p z^{-T} G'_y] \]
\[ = H'_y (I + H'_y G'_y \bar{A}_y z^{-T} (I + G'_y \bar{H}_y^{-1} A_y C_y G'_y)^{-1} \bar{A}_y) \]
\[ - H'_y G'_y \bar{A}_p z^{-T} (I + G'_y \bar{H}_y^{-1} A_y C_y G'_y)^{-1} G'_y H'_y^{-1} \]  

(97)

If it is assumed that the spectral factor \( D_y \) is strictly Schur, because of the disturbance and noise model definitions, and it is also assumed an asymptotically stable closed loop solution is required, then noting (86) the term \( T'_y \) must be asymptotically stable. It may also be noted for later use, from (90), that the term \( T'_s \) is strictly unstable.

A1.6 Tracking equations

The diophantine equation (84) may be utilized again to simplify the tracking cost term (Sebek and Kucera 1982 [23]), which may be written as:
\[ Y^{T}_1 \bar{Q}_y W_y W'_y Y^{T}_1 = (G_y \bar{A}_y z^{-T} + z^{T} D^{T}_y F'_p) z^{-T} D_y C_y D^{T}_y \]
\[ = G_y \bar{A}_y z^{-T} D_y C_y D^{T}_y + z^{T} D^{T}_y F'_p z^{-T} D_y C_y D^{T}_y \]  

(98)

To further simplify the first term in this expression recall that \( C_y = \bar{C}_y D^{T}_y \) and introduce the filtering diophantine equation:
\[ z^{-\varepsilon_1}G_{i,0}D_{i,0}^p + \mathbf{A}_yF_{i,0}^p = D_{i,1}C_{i,0}^p z^{-\varepsilon_1} \]  

(99)

It follows that part of the cost term may now be written as:

\[ Y_{\varepsilon_1}^{-1}(\mathbf{Q}_yW_y Y_{\varepsilon_1}^{-1})^{-1} = G_{i,0}^pD_{i,0}^{-1}z^{-\varepsilon_1}G_{i,0}^{-1} + (G_{i,0}^p z^{-\varepsilon_1}F_{i,0}^p D_{i,0}^{p-1} z^{-\varepsilon_1}) \]

(100)

**A1.7 Tracking cost terms and \( T_r^+ \) and \( T_r^- \)**

The tracking cost term therefore follows from (68) and (100) as:

\[ T_r = Y_y S_y^r Y_y^{-1} - Y_{\varepsilon_1}^{-1}(\mathbf{Q}_yW_y Y_{\varepsilon_1}^{-1})^{-1} = [D_{i,0}^p \mathbf{A}_y D_{i,0}^{-1} - G_{i,0}^p \mathbf{A}_y z^{-\varepsilon_1} G_{i,0}^{-1}] \]

\[ - D_{i,0}^{p-1}(D_{i,0}^pG_{i,0}^p z^{-\varepsilon_1}F_{i,0}^p D_{i,0}^{p-1} z^{-\varepsilon_1} + F_{i,0}^p z^{-\varepsilon_1} D_{i,1}C_{i,0}^p z^{-\varepsilon_1})D_{i,0}^{p-1} \]

(101)

where \( T_r = T_r^+ - T_r^- \) and the stable \( T_r^+ = [\cdot] \) and unstable \( T_r^- = D_{i,0}^{p-1}(D_{i,0}^p) \) terms may also be identified from this expression.

**A1.8 Simplification of cost terms and minimisation**

From (97) and (100) the total cost term may be written in the simpler form:

\[ I_c = \text{trace}(T_r T_r^+) + \text{trace}(T_r^+ T_r^-) + I_{\alpha 0} \]

\[ = \text{trace}((T_r^+ - T_r^-)(T_r^- - T_r^-)^\dagger) + I_{\alpha 0} \]

\[ + \text{trace}([T_r^- - T_r^-]([T_r^- - T_r^-]^\dagger) + I_{\alpha 0}) \]

(102)

The minimisation of this cost term may follow a standard complex integral minimisation procedure explained in Grimble (2001 [25]). The complex-integral of the cross-terms in the above expression may easily be shown to be null and the cost minimization problem therefore involves:

\[ J = \frac{1}{2\pi j} \int (\text{trace}(T_r T_r^+) + T_r^- T_r^-)^\dagger \]

\[ + \text{trace}([T_r^- - T_r^-]([T_r^- - T_r^-]^\dagger) + I_{\alpha 0}) \frac{dz}{z} \]

(103)

The terms \( I_{\alpha 0}, T_r^+ \) and \( T_r^- \) are independent of the choice of controller terms. It follows that the full-order optimal control must satisfy: \( T_r^+ = 0 \) and \( T_r^- = 0 \) and the minimum cost then becomes:

\[ J_{\text{min}} = \frac{1}{2\pi j} \int (\text{trace}(T_r T_r^+) + \text{trace}(T_r^- T_r^-) + I_{\alpha 0}) \frac{dz}{z} \]

(104)

**A1.9 Definition of gains and sensitivities**

Expressions may be simplified by defining various control and filter gains as:

\[ K_c = H_{i,0}^{-1}G_{i,0}^p, \quad K_{f,1} = G_{i,0}^p H_{i,0}^{-1}, \quad K_{f,2} = G_{i,0}^p H_{i,0}^{-1} \]

(105)

Various return-difference and sensitivity relationships may be established, using these gains, as follows.

From the control implied equation (83):

\[ H_{i,0}^{-1}D_{i,0}^p \mathbf{A}_y^j = I + H_{i,0}^p G_{i,0}^p D_{i,0}^{p-1} \]

(106)

Note that the filtering implied equation (86) gives a similar result:

\[ A_{i,0}^p D_{i,0}^p H_{i,0}^{-1} = I + A_{i,0}^p \mathbf{C}_y G_{i,0}^p H_{i,0}^{-1} = I + W_r K_{f,1} = S_{i,1}^j \]

(107)

and the second filtering implied equation (89) gives:

\[ A_{i,0}^p D_{i,0}^p H_{i,0}^{-1} = I + A_{i,0}^p \mathbf{C}_y G_{i,0}^p H_{i,0}^{-1} = I + W_r K_{f,2} = S_{i,2}^j \]

(108)

**A1.10 Full-order optimal controller expressions**

The regulating loop feedback controller follows from (97) by setting \( T_r^+ \) to zero, and using the gain definitions (105), to obtain:

\[ K_i = [I + K_c(z \mathbf{A}_y^j + K_{f,2} C_{i,0})^{-1} B]^{-1} K_c(z \mathbf{A}_y^j + K_{f,2} C_{i,0})^{-1} K_j \]

\[ = K_c(z \mathbf{A}_y^j + K_{f,2} C_{i,0})^{-1} [I + W_r K_{f,2}]^{-1} \]

(109)

The tracking controller follows from (101) by setting the tracking term \( T_r^- \) to zero and using (108) to obtain:

\[ K_s = S_{i,0} H_{i,0}^{-1} G_{i,0}^p \mathbf{A}_y^j z^{-\varepsilon_1} G_{i,0}^p D_{i,0}^{p-1} \mathbf{A}_p \]

\[ = S_{i,0} K_s z^{-\varepsilon_1} K_{r,0} S_{i,0} \]

\[ = [I + K_c(z \mathbf{A}_y^j + K_{f,1} C_{i,0})^{-1} B]^{-1} \]

\[ \times K_c(z \mathbf{A}_y^j + K_{f,1} C_{i,0})^{-1} K_{r,0} \]

\[ = [I + K_c(z \mathbf{A}_y^j + K_{f,1} C_{i,0})^{-1} B]^{-1} \]

\[ \times K_c(z \mathbf{A}_y^j + K_{f,1} C_{i,0})^{-1} K_{r,0} \]

(111)

**A1.11 Return-difference and sensitivity relationships**

First denote the square bracketed term in (111) as:

\[ S_{i,2}^j = [I + K_c(z \mathbf{A}_y^j + K_{f,1} C_{i,0})^{-1} B] \]
Note the following result for the return-difference $S^{-1} = I + K_1 W$ matrix:

\[
(I + K_1 W)^{-1} = [I + K_1 (\bar{z} \bar{p} + K_1 C_2)]^{-1} B
+ K_1 (\bar{z} \bar{p} + K_1 C_2)^{-1} K_1 W
\]

\[
= S_{o}(I + K_1 (\bar{z} \bar{p} + K_1 C_2))^{-1}
\times (I + K_1 (\bar{z} \bar{p} + K_1 C_2)\bar{z}^{-1} B)
\]

\[
= S_{o}(I + \bar{A}_p \bar{z}^{-1} B) = S_{o} S_{S_{r}}^{-1}
\]

and the relationship between the sensitivities:

\[
S = S_{o} S_{S_{r}}^{-1}
\]  

(112)

A.1.2 Restricted Structure optimization

The terms $T_r^*$ and $T_o^*$ are null for an optimal solution. However, if the controller structure is restricted the effective variance of the output of the correspondence terms in (104) must be minimised. That is, the terms to be minimised involve the following operators:

\[
K_1 (\bar{z} \bar{p} + K_1 C_2)^{-1} K_1 W
\]

\[
\times (I + K_1 (\bar{z} \bar{p} + K_1 C_2)\bar{z}^{-1} B)
\]

\[
= S_{o}(I + \bar{A}_p \bar{z}^{-1} B) = S_{o} S_{S_{r}}^{-1}
\]

The numerator includes the unknown gains ($k_0, k_1, k_2$) to be optimised and the denominator is assumed to be known in this filtered PID control case.

Let the superscripts $r$ and $i$ denote the real and imaginary parts of a complex function, so that:

\[
C_{ai} = C_{ar}^* + jC_{ri}^* \quad \text{and} \quad C_{ri} = C_{ar}^* + jC_{ri}^*
\]  

(118)

The numerator term may be split into complex frequency dependent components, through comparison with (116)

\[
C_{ai} = k_0 \alpha_i (\bar{z}^{-1}) + k_i \alpha_i (\bar{z}^{-1}) + k_2 \alpha_i (\bar{z}^{-1})
\]

\[
= C_{ai} + jC_{ri}
\]

where $\alpha_0 = (1 - \bar{z}^{-1})(1 - \alpha_1 z^{-1}), \alpha_1 = (1 - \alpha_2 z^{-1}), \alpha_2 = (1 - z^{-1})$ and hence,

\[
C_{ai} = k_0 \alpha_i + k_i \alpha_i + k_2 \alpha_i \quad \text{and} \quad C_{ri} = k_0 \alpha_i + k_i \alpha_i + k_2 \alpha_i
\]  

(119)

In deriving the following expressions account is taken of the fact that the solution of the optimization problem is to be found by iteration (Luenberger 1969 [26]). The denominator term in $T_r^*$ will therefore be assumed known, and the minimisation, at the first iteration, will be performed on the numerator terms. Thus let,

\[
T_r^* = C_{ai} L_{si} - C_{ri} L_{ri}
\]

where $L_{si}$ and $L_{ri}$ are defined as:

\[
L_{si} = L_1 (C_{ai} L_4 + C_{ri} L_4) \quad \text{and} \quad L_{ri} = L_2 (C_{ai} L_4 + C_{ri} L_4)
\]  

(120)

Substituting from equations (118) and (120)

\[
T_r^* = (C_{ar}^* + jC_{ri}^*) (L_{si} + jL_{ri}) - (C_{ar}^* + jC_{ri}^*) (L_{si} + jL_{ri})
\]

\[
= C_{ar}^* L_{ri} - C_{ri}^* L_{ri} + C_{ri}^* L_{si} + C_{ri}^* L_{ri}
\]

and after substitution from (119) obtain

\[
T_r^* = k_0 \left( (\alpha \bar{L}_{si} - \alpha \bar{L}_{ri}) + j(\alpha \bar{L}_{ri} + \alpha \bar{L}_{si}) \right)
\]

(115)

so that

\[
C_{ai} = k_0 (1 - \bar{z}^{-1})(1 - \alpha z^{-1}) + k_i (1 - \bar{z}^{-1})
\]

\[
+ k_2 (1 - \bar{z}^{-1})
\]

(116)

and

\[
C_{ri} = (1 - \bar{z}^{-1})(1 - \alpha z^{-1})
\]

(117)

The numerator includes the unknown gains ($k_0, k_1, k_2$) to be optimised and the denominator is assumed to be known in this filtered PID control case.

The numerical algorithm to compute the restricted structure controller can be summarized as follows: Assume that $K_1$ has a modified PID structure of the form:

\[
K_1 = k_0 + k_1 (1 - \bar{z}^{-1}) + k_2 (1 - \bar{z}^{-1})(1 - \alpha z^{-1})
\]

(115)
where the integral is approximated as:

$$\int \approx \sum_{n} \left[ \frac{C_p L_2^n - C_0 L_2^n}{C_0 L_2^n + C_p L_2^n} \right]$$

The real and imaginary parts of $T^*_e$ may be written as:

$$T^*_{re} = T^*_r + T^*_i$$
and it follows that,

$$\sqrt{\|T^*_{re}\|^2 + \|T^*_{ri}\|^2}$$

Write a vector form of the above equations that will enable the optimization to be performed, with respect to the unknown gains $k_0, k_1, k_2$, as:

$$T^*_{re} = \begin{bmatrix} T^*_{re} \\ T^*_{ri} \end{bmatrix}$$

where

$$F = \begin{bmatrix} (\alpha^2 L_n^0 - \alpha L_n^1) (\alpha L_n^0 - \alpha^2 L_n^1) \\ (\alpha L_n^0 + \alpha^2 L_n^1) (\alpha L_n^0 + \alpha^2 L_n^1) \end{bmatrix}$$

and

$$L = \frac{C_0 L_2^n - C_0 L_2^n}{C_0 L_2^n + C_p L_2^n}$$

The cost-function can be optimised directly but a simple iterative solution can be obtained if the integral is approximated by a summation with a sufficient number of frequency points $\{\omega_0, \omega_1, \ldots, \omega_N\}$. The optimisation can then be performed by minimising the sum of squares at each of the frequency points as used in a different context by Yukitomo et al (1998 [27]). The maximum frequency can be taken as the foldover frequency. The minimization of the cost term $J_d$ is therefore required, where the integral is approximated as:

$$J_d = \sum_{i=1}^{N} (F x - L)^T (F x - L) = (b - Ax)^T (b - Ax)$$

where

$$A = \begin{bmatrix} F(z^{-j \omega_0 \pi}) \\ \vdots \\ F(z^{-j \omega_N \pi}) \end{bmatrix}, \quad b = \begin{bmatrix} L(z^{-j \omega_0 \pi}) \\ \vdots \\ L(z^{-j \omega_N \pi}) \end{bmatrix}, \quad x = \begin{bmatrix} k_0 \\ k_1 \\ k_2 \end{bmatrix}$$

Assuming the matrix $A^T A$ is not singular the least squares optimal solution follows as:

$$x = (A^T A)^{-1} A^T b$$

Since in practice the denominator of the term $T^*_e$ is a function of these gains iteration is required to successively perform the gain calculation and the calculation of this denominator term. Since this optimization problem is non-linear (the unknown gains appear in the denominators of certain transfers) there may not be a unique minimum. However, the algorithm presented below does always appear to converge to an optimal solution in many industrial and academic examples.

**Iterative Regulating Loop Benchmark Computation**

The following successive approximation algorithm (Luenberger 1969 [26]) can be used to compute the restricted structure $H_2$ optimal control and benchmark cost.

**Step. Restricted Structure $H_2$ Feedback Control Benchmark**

1. Solve for the spectral factors (25) and (26), and the diophantine equations (28) to (31). Compute full order feedback controller $K(z^{-1})$ by choosing appropriate weightings and solving (35).

2. For restricted structure PID let $\alpha_0 = (1 - z^{-1})(1 - \theta^2), \alpha_1 = (1 - \theta^2), \alpha_2 = (1 - z^{-2})^2$.

3. Define the polynomials:

$$L_1 = H(z^{-1})[1 + K(z^{-1})], \quad L_2 = H(z^{-1})K(z^{-1})D(z^{-1})$$

4. Initialise $k_0, k_1, k_2$.

5. Compute

$$C_0(z^{-1}) = \alpha_0(z^{-1})k_0 + \alpha(z^{-1})k_1 + \alpha(z^{-1})k_2$$

and

$$C_0(z^{-1}) = \alpha(z^{-1})k_0$$

6. Compute

$$L_0(z^{-1}) = L_1(C_0 L_3 + C_0 L_4)^{-1}$$

and

$$L_0(z^{-1}) = L_2(C_0 L_3 + C_0 L_4)^{-1}$$

7. Compute for all chosen frequencies $L_{i0}(\omega), L_{i1}(\omega), L_{i2}(\omega), \alpha_{i0}(\omega), \alpha_{i1}(\omega), \alpha_{i2}(\omega), \alpha_{i3}(\omega), \alpha_{i4}(\omega), \alpha_{i5}(\omega)$, and hence find:

$$f_{i0}(\omega) = \alpha_{i0}(\omega)L_{i0}(\omega) - \alpha_{i0}(\omega)L_{i0}(\omega)$$

$$f_{i1}(\omega) = \alpha_{i1}(\omega)L_{i1}(\omega) - \alpha_{i1}(\omega)L_{i1}(\omega)$$

$$f_{i2}(\omega) = \alpha_{i2}(\omega)L_{i2}(\omega) - \alpha_{i2}(\omega)L_{i2}(\omega)$$

$$f_{i3}(\omega) = \alpha_{i3}(\omega)L_{i3}(\omega) - \alpha_{i3}(\omega)L_{i3}(\omega)$$

$$f_{i4}(\omega) = \alpha_{i4}(\omega)L_{i4}(\omega) - \alpha_{i4}(\omega)L_{i4}(\omega)$$

$$f_{i5}(\omega) = \alpha_{i5}(\omega)L_{i5}(\omega) - \alpha_{i5}(\omega)L_{i5}(\omega)$$

8. Compute for all chosen frequencies $C_{i0}(\omega), C_{i1}(\omega), C_{i2}(\omega), C_{i3}(\omega)$, and find:

$$L_{i0}(\omega) = C_{i0}(\omega)L_{i0}(\omega) - C_{i0}(\omega)L_{i0}(\omega)$$

$$L_{i1}(\omega) = C_{i1}(\omega)L_{i1}(\omega) - C_{i1}(\omega)L_{i1}(\omega)$$

$$L_{i2}(\omega) = C_{i2}(\omega)L_{i2}(\omega) + C_{i2}(\omega)L_{i2}(\omega)$$
(9) \[ A_s = \begin{bmatrix} f_1^*(\omega_0) & f_2^*(\omega_0) & f_3^*(\omega_0) \\ \vdots & \vdots & \vdots \\ f_1^*(\omega_N) & f_2^*(\omega_N) & f_3^*(\omega_N) \end{bmatrix}, L_s = \begin{bmatrix} L_1(\omega_0) \\ \vdots \\ L_1(\omega_N) \end{bmatrix} \]

(10) If error \( e > \) go to (3), else compute the restricted structure feedback controller:

\[ C_{\alpha}(z^{-1}) = \alpha_\delta(z^{-1})k_x + \alpha_\gamma(z^{-1})k_y + \alpha_\zeta(z^{-1})k_z \]

and:

\[ K_{\alpha}(z^{-1}) = C_{\alpha}(z^{-1})C_{\alpha}(z^{-1})^{-1}. \]

(11) Evaluate benchmark cost values, using the results in Theorem 3.2, for the restricted structure optimal controller.

### Tracking control Benchmark cost computation

The computation of the tracking controller follows steps very similar to the above. However, iteration is not required, since the unknown gains only appear linearly (in the numerator of the cost expression).

#### Step. Restricted structure \( H_2 \) tracking control benchmark

1. Calculate the spectral factor \( D_{10} \) from (27) and the diophantine equation solutions (32) and (33). Compute the full order tracking controller: \( K_{\delta}(z^{-1}) \) from (35).

2. For restricted structure PID let \( \alpha_\delta = (1 - z^{-1})(1 - \sigma z) \), \( \alpha_\gamma = (1 - \sigma z)^2 \), \( \alpha_\zeta = (1 - z^{-1})^2 \) and \( C_{\alpha}(z^{-1}) = \alpha_\delta(z^{-1}) \).

3. (Define)

\[ L_s = H_s[I + K, S, B]A_s^{-1}D_{10}C_{\alpha}^{-1} \]

\[ L_s = \begin{bmatrix} G_{11}A_{11}^{-1} \end{bmatrix} \]

4. Compute for all chosen frequencies \( L_1(\omega_0) \), \( L_1(\omega_N) \), \( L_2(\omega_0) \), \( L_3(\omega_N) \), \( \alpha_\delta(\omega_0) \), \( \alpha_\gamma(\omega_0) \), \( \alpha_\zeta(\omega_0) \), \( \alpha_\delta(\omega_N) \), \( \alpha_\gamma(\omega_N) \), and hence find:

\[ f_1^*(\omega_0) = \alpha_\delta(\omega_0)L_1(\omega_0) + \alpha_\gamma(\omega_0)L_1(\omega_0), \]

\[ f_2^*(\omega_0) = \alpha_\delta(\omega_0)L_1(\omega_0) + \alpha_\gamma(\omega_0)L_1(\omega_0) \]

\[ f_3^*(\omega_0) = \alpha_\delta(\omega_0)L_1(\omega_0) + \alpha_\gamma(\omega_0)L_1(\omega_0), \]

\[ f_1^*(\omega_N) = \alpha_\delta(\omega_N)L_1(\omega_N) + \alpha_\gamma(\omega_N)L_1(\omega_N), \]

\[ f_2^*(\omega_N) = \alpha_\delta(\omega_N)L_1(\omega_N) + \alpha_\gamma(\omega_N)L_1(\omega_N) \]

(5) Compute for all chosen frequencies \( C_{\alpha}(\omega_0) \), \( C_{\alpha}(\omega_N) \), \( C_{\alpha}(\omega_0) \), \( C_{\alpha}(\omega_N) \), and find:

\[ L_1(\omega_0) = C_{\alpha}(\omega_0)L_{10}(\omega_0) - C_{\alpha}L_{10}(\omega_0) \]

\[ L_1(\omega_N) = C_{\alpha}(\omega_N)L_{10}(\omega_N) + C_{\alpha}L_{10}(\omega_N) \]

(6) \[ A_s = \begin{bmatrix} f_1^*(\omega_0) & \cdots & f_3^*(\omega_0) \\ f_1^*(\omega_N) & \cdots & f_3^*(\omega_N) \end{bmatrix}, \quad B_s = \begin{bmatrix} L_1(\omega_0) \\ \vdots \\ L_1(\omega_N) \end{bmatrix} \]

(7) \[ [k_1, k_2, k_3]^T = \left[A_s^T A_s\right]^{-1} A_s^T B_s \]

(8) Compute the restricted structure tracking controller:

\[ C_{\alpha}(z^{-1}) = \alpha_\delta(z^{-1})k_x + \alpha_\gamma(z^{-1})k_y + \alpha_\zeta(z^{-1})k_z \]

and:

\[ K_{\delta}(z^{-1}) = C_{\alpha}(z^{-1})C_{\alpha}(z^{-1})^{-1}. \]

(9) Compute benchmark cost values, using the results in Theorem 3.2, for the restricted structure optimal tracking controller.

### MATHEMATICAL PRELIMINARIES FOR DISCRETE-TIME SYSTEMS

The mathematical objects considered are real rational polynomial matrices in the indeterminate \( z^{-1} \). For any matrix \( A \), let \( A^* \) and \( \text{trace} \{A\} \) denote the transpose and trace of the matrix \( A \), respectively. The polynomial matrix \( D \) will be called Schur if its inverse is analytic outside the unit-circle \( |z| > 1 \) and strictly Schur if its inverse is analytic outside \( |z| \geq 1 \). For the adjoint of any real rational matrix \( W(z^{-1}) \), write \( W^*(z^{-1}) = W^T(z) \).

#### SYMBOLS AND NOTATION

- \( \delta_{it} \) Kronecker delta function \( \delta_{it} = 1 \) for \( t = i \) and \( \delta_{it} = 0 \) for \( t \neq i \).
- \( z^{-1} \) The unit delay operator \( z^{-1}x(t) = x(t - 1) \) or the \( Z \) transfer-function complex-number inverse (this should be clear according to the context of its use).
- \( D^* \) Adjoint of the polynomial matrix \( D(z^{-1}) \), where \( D = D^*(z) \).
- \( I_q \) Identity matrix of dimension \( q \times q \).
- \( E_{11} \) Ensemble average.
- \( R^* \) Euclidean space of n-dimensional real-vectors.
**REFERENCES**

Michael Grimble was born in Grimsby, England. He acquired a BSc (Coventry), MSc, PhD and DSc degrees from the University of Birmingham. In 1981, The University of Strathclyde, Glasgow, appointed him to the Professorship of Industrial Systems and he is now the Director of the Industrial Control Centre. His Centre is concerned with industrial control problems, particularly those arising in the Aerospace, Manufacturing, Process Control, Wind adaptive control, $H_\infty$ robust control theory, multivariable design techniques, optimal control and estimation theory. He is the Managing Editor of the International Journal of Adaptive Control and Signal Processing published by John Wiley Ltd. He is also the Managing Editor of the Wiley International Journal of Robust and Non-linear Control. He was the editor of the Prentice Hall International series of books on Systems and Control Engineering and also the Prentice-Hall series on Acoustics Speech and Signal Processing. He is managing editor of the Springer Verlag Monograph Series on Advances in Industrial Control and is an Editor of the Springer Lecture Notes in Control and Information Sciences series. He is also a managing editor of the Springer Verlag text book series on Control and Signal Processing.

The Institution of Electrical Engineers presented him with the Heaviside Premium in 1978 for his papers on control engineering. The following year, 1979, he was awarded jointly the Coopers Hill War Memorial Prize and Medal by the Institutions of Electrical, Mechanical and Civil Engineering. The Institute of Measurement and Control awarded him the 1991 Honeywell International Medal. He was awarded an IEEE Fellowship in 1992 and he was recognised at the 1993 Edinburgh International Science Festival as one of Scotland’s four most cited Scientists. He was appointed a Fellow of the Royal Society of Edinburgh in 1999.