Brief Paper

2-D ALGEBRAIC TEST FOR ROBUST STABILITY OF QUASIPOLYNOMIALS WITH INTERVAL PARAMETERS

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ABSTRACT

Determining the robust stability of interval quasipolynomials leads to a NP problem: an enormous number of testing edge polynomials. This paper develops an efficient approach to reducing the number of testing edge polynomials. This paper solves the stability test problem of interval quasipolynomials by transforming interval quasipolynomials into two-dimensional (2-D) interval polynomials. It is shown that the robust stability of an interval 2-D polynomial can ensure the stability of the quasipolynomial, and the algebraic test algorithm for 2-D $s-z$ interval polynomials is provided. The stability of 2-D $s-z$ vertex polynomials and 2-D $s-z$ edge polynomials were tested by using a Schur Table of complex polynomials.

KeyWords: Time-delay systems, interval parameters, robust stability, quasipolynomials with interval parameters, 2-D $s-z$ polynomials, edge test set, algebraic test theorems.

I. INTRODUCTION

The characteristic polynomials of interval time-delay systems are interval quasipolynomials, and the stability of interval time-delay systems can ensure the stability of the corresponding interval quasipolynomials [1-6]. In 1989, the authors of [1] revealed that the robust stability of interval quasipolynomials can be guaranteed by the stability of the exposed edge family of quasipolynomials, but the authors did not describe the approach to identifying the exposed edge family of quasipolynomials, which means that for interval quasipolynomials of order $M \times N$, there are $2^M (M+1)^N$ edge quasipolynomials that have to be tested, which is a NP problem. To reduce the size of the edge family of quasipolynomials to be tested, the authors of [2] proposed a necessary phase condition to construct the test set of the edge family of quasipolynomials, but obtaining the edge set also requires $\binom{2^M (M+1)^N}{2}$ computations, and this is also a NP problem. Meanwhile, the authors of [2] suggested that the Kharti- onov extreme test may be applied to interval quasipolynomials. Unfortunately, the authors did not explore deeper. Recently, the authors of [3-5] independently investigated the Kharti- onov extreme test, like interval quasipolynomials, and proposed two different algorithms for the edge test of interval quasipolynomials. The main difference in their approaches was that a 2-D technique was adopted in [4,5], while in [3], a classical 1-D tool was used. In fact, the results in [3] can be obtained by using the approach in [4,5], and this paper will include some related discussion. This paper will further the 2-D work reported in [4,5] and focus on the construction of an edge quasipolynomial family. Because the zeros of a 2-D polynomial that is obtained from a quasipolynomial are located in the hybrid $s-z$ domain [4,5], which is similar to the 2-D continuous-discrete systems [8,9], most of the reported 2-D robust stability results [10-12,15] can not be directly applied. If we regard quasipolynomials as a special kind of 2-D hybrid polynomials, we can solve the problem of the robust stability of quasipolynomials in the 2-D $s-z$ domain. The authors of [4] and [5] showed that the results of in [14] for the stability of quasipolynomials with fixed parameters were incorrect; generally, the stability of a 2-D polynomial is a sufficient condition for the corresponding quasipolynomial instead of a necessary and sufficient one as claimed in [14]. The results reported in this
paper are also sufficient conditions for interval quasipolynomials. This paper extends the stability results in [13] for time-delay systems with fixed parameters to the robust stability of time-delay systems with interval parameters. We will show that quasipolynomials are a special case of 2-D s-z interval hybrid polynomials. Based on our results, we will develop a 2-D algebraic approach for testing the stability of quasipolynomials. We will show that the stability of the 2-D form of characteristic polynomials of systems can ensure the stability of the corresponding quasipolynomials.

II. INTERVAL TIME-DELAY SYSTEMS AND QUASIPOLYNOMIALS

Time-delay systems with interval parameters can be defined by means of a recursive model. We will first discuss the recursive model of time-delay systems. The recursive model is

\[ y(t, a, b) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{mn} \frac{\partial^n u(t+\tau_n)}{\partial t^n} - \sum_{m=0}^{M} \sum_{n=0}^{N} b_{mn} \frac{\partial^m y(t-\tau_m, a, b)}{\partial t^m}. \]  

(1)

where \( \tau_n \) are constants; \( \tau_n \geq 0 \), \( a = [a_{mn}] \), \( a_{mn} = [a_{mn}^-, a_{mn}^+] \), \( b = [b_{mn}] \), \( b_{mn} = [b_{mn}^-, b_{mn}^+] \), \( a_{mn}^- \) and \( a_{mn}^+ \) are the lower bounds and upper bounds of the uncertain coefficients \( a_{mn} \), respectively, and so are \( b_{mn}^- \) and \( b_{mn}^+ \). \( u(t) \) and \( y(t, a, b) \) are the input signal and output signal of the time-delay system, respectively.

If \( \tau_n = nT \) in Eq. (1), the systems are considered commensurate time delay systems [3]. In this paper, we will mainly study this class of systems, though some results have no such limitation for time delay.

Applying Laplace transform to Eq. (1), we get the transfer function of the time-delay system as follows:

\[ H(s, a, b) = \frac{\sum_{n=0}^{N} \sum_{m=0}^{M} a_{mn} s^m e^{-\tau_n s}}{\sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} s^m e^{-\tau_m s}}. \]  

(2)

System (1) has the following characteristic polynomial:

\[ B(s, b) = \sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} s^m e^{-\tau_n s}. \]  

(3)

Remark. The characteristic polynomial (3) is directly derived from the recursive time delay equation (1); thus, there is no multiplication of interval parameters \( b \). However, if the characteristic polynomial (3) is obtained from state-space equation in [1], then there will be multiplication of interval parameters \( b \). This paper will only examine time-delay systems in the form of Eq. (1) to avoid the problem of multiplying the interval parameters of state-space time-delay systems. To date, the problem of multiplying the interval parameters of state-space time-delay systems is still unsolved; it is difficult to solve by using the approach of this paper. In fact, it was incorrect for the authors of [1] to introduce quasipolynomials from state-space time-delay systems, since the interval parameters from system matrix to characteristic polynomial are not a simple linear affining.

The polynomials in Eq. (3) are called quasipolynomials [1-6]. The asymptotic stability of system (1) can be ensured by the stability of quasipolynomials (3). Let

\[ b_n(s, b) = \sum_{m=0}^{M} b_{mn} s^m. \]  

(4)

We can express the quasipolynomial (3) as a composite polynomial as follows:

\[ B(s, b) = \sum_{n=0}^{N} b_n(s, b) e^{-\tau_n s}, \quad \tau_n = nT. \]  

(5)

Definition 1. If the quasipolynomial (5) satisfies

\[ B(s, b) = \sum_{n=0}^{N} b_n(s, b) e^{-\tau_n s} \neq 0, \quad \text{Re} s \geq 0, \]  

(6)

then the quasipolynomial is Hurwitz stable.

Since the polynomial in Eq. (6) has an infinite number of roots, it is difficult to develop an algebraic algorithm in the 1-D s domain for the stability of the quasipolynomials in Eq. (6) [4,5]. That is why in this paper, we will use a 2-D algorithm to test the stability of quasipolynomials. The following theorem changes the 1-D stability test for quasipolynomials (6) into a 2-D one under the constraint of time delay.

Theorem 1. If there is a constant \( T \) such that \( \tau_n = nT \) for quasipolynomial (5), such that

\[ B(s, z, b) = \sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} s^m z^{-n} \neq 0 \]  

(7)

for \( \text{Re} s \geq 0 \) and \( |z| \geq 1 \), where the coefficients \( b_{mn} \) are those of quasipolynomials, which are given by Eq. (3), then the corresponding quasipolynomial (5) is stable.

Proof. Let \( z = e^{sT} \); due to the assumption that there is a constant \( T \) such that \( \tau_n = nT \); we have

\[ z^{-n} = e^{-nT} = e^{-\tau_n s}. \]  

(8)

Due to Eq. (8), it is easy to show that \( |z| \geq 1 \) when \( \text{Re} s \geq 0 \). Substituting Eq. (8) into Eq. (7), we get

\[ B(s, b) = \sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} s^m e^{-\tau_n s} \neq 0, \quad \text{for} \text{Re} s \geq 0. \]  

(9)
Remarks.
(1) Theorem 1 is especially valid for the following time-delay systems:
\[ y(t, \mathbf{a}, \mathbf{b}) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_m \frac{\partial^m u(t-\tau T)}{\partial t^m} - \sum_{m=0}^{M} \sum_{n=0}^{N} b_m \frac{\partial^m y(t-\tau T, \mathbf{a}, \mathbf{b})}{\partial t^m}, \]
where \( T \) is a nonnegative constant, and \( u(t) \) and \( y(t) \) are the input signal and output signal of the time-delay system, respectively. It is easy to find that the quasipolynomial (9) is the characteristic polynomial of the system (10).
(2) The 2-D condition (7) of Theorem 1 for the stability of the quasipolynomial (9) is not necessary, since the quasipolynomial in Eq. (9) or Eq.(9) is only a subset of the 2-D polynomial (7) in the 2-D \( s-z \) domain. The authors of [14] wrongly stated that the quasipolynomial in Eq. (9) is equal to a 2-D polynomial (7).
(3) The 2-D condition (7) is sufficient for condition (9). From Eq. (8) of the proof of Theorem 1, we can see that \( z = e^{-\tau T} \) is only a subregion of in 2-D \( s-z \) domain, so the quasipolynomial (9) is a special case of the 2-D \( s-z \) polynomial in Eq. (7). When condition (8) is satisfied, the 2-D \( s-z \) polynomial in Eq. (7) changes back to the quasipolynomial (9).
(4) Sometimes, \( \tau \neq kT \) in the system (1), so we cannot apply Theorem 1 directly. The authors of [4] and [5] provided a theorem that can be used to solve the above problem for a more general class of time-delay systems. Due to space limitations, we will focus our discussion on the class of interval quasipolynomials with commensurate time-delay.

III. THE EDGE TEST FOR INTERVAL 2-D POLYNOMIALS

Definition 2. Suppose that \( B_i(s, z) \) and \( B_j(s, z) \) are vertex polynomials of the interval 2-D polynomial in Eq.(5). Then, the edge of the interval bivariate polynomial is defined as
\[ E_{ij}(s, z) = x B_j(s, z) + (1-x) B_i(s, z); \]
for all \( x \in (0, 1), i, j \in \{1, ..., K\}, i \neq j, K = 2^{(M+1)(N+1)} \) is the number of vertex polynomials of the interval 2-D polynomials.

The classical test for quasipolynomials tests every edge of \( b_A(s, \mathbf{b}) \) according the vertex points and the edges of \( \mathbf{b} \), since there is no way to identify the exposed edges of polynomial set \( b_A(s, \mathbf{b}) \). Thus, the number of tested edges will be \( 2^{(M+1)(N+1)} \), which is a NP problem. In this paper, we will try to solve this problem.

Property 1. The coefficients of \( B(s, z, \mathbf{b}) \) in Eq. (11) are linear affine mappings of its parameters for an arbitrary but fixed complex variable \( s \).

Proof. For an arbitrary but fixed complex variable \( s \) and two arbitrary points \( \mathbf{b} \) and \( \mathbf{b}^1 \) in the parameter set \( \mathbf{b} \) defined by Eq. (4), we have
\[ B(s, z, \mathbf{b}) = \sum_{n=0}^{N} b_n(s, \mathbf{b}) z^n. \]
It is easy to see that the coefficients in Eq. (12) satisfy
\[ b_n(s, \mathbf{b}^0 + \mathbf{b}^1) = \sum_{n=0}^{N} (b_{n0} + b_{n1}) z^n = \sum_{n=0}^{N} b_{n0} z^n + \sum_{n=0}^{N} b_{n1} z^n = b_n(s, \mathbf{b}^0) + b_n(s, \mathbf{b}^1). \]
Thus, \( b_n(s, \mathbf{b}), n = 0, 1, ..., N \), are linear affine mappings of its parameters in the \( s \) domain.

The linear affine property of \( b_n(s, \mathbf{b}) \) and the coefficients of \( B(s, z, \mathbf{b}) \) can ensure that the vertices, edges, and faces of \( b_A(s, \mathbf{b}) \) are mappings of those of parameter set \( \mathbf{b} \). Thus, we can derive an edge test for the robust Hurwitz-Schur stability of interval bivariate polynomials. Since the vertex test is not valid for the problem of the Hurwitz-Schur stability of interval 2-D polynomials [8,9], generally, the edge test is used to solve the problem. From Property 1, we can define the edges of interval 2-D polynomials.

The following theorem provides an approach to expressing 2-D \( s-z \) polynomials as 1-D real coefficient polynomials and 1-D complex variable coefficient polynomials [8,9].

Theorem 2. [8,9] An interval 2-D bivariate polynomial is Hurwitz-Schur stable if and only if
\begin{align}
(1) \quad & B(s, 1, \mathbf{b}) = \sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} s^n \neq 0, \quad \text{for } \Re s \geq 0, \quad (13) \\
(2) \quad & B(j\omega, z, \mathbf{b}) = \sum_{n=0}^{N} \sum_{m=0}^{M} b_{mn} (j\omega)^n z^{-n} \neq 0, \quad \text{for } \omega \in \mathbb{R} \text{ and } |\omega| \geq 1. \quad (14)
\end{align}
Condition (1) in Theorem 2 transforms the 2-D polynomial \( B(s, z, \mathbf{b}) \) in Eq. (7) into a 1-D interval one, and we can apply the following theorem to test its stability.

Theorem 3. Condition (1) of Theorem 2 is true if and only if
\[ B(s, 1, \mathbf{b}) = \sum_{m=0}^{M} u_m s^n \neq 0, \quad \text{for } \Re s \geq 0, \quad (15) \]
where \( u_n \in [u_n^u, u_n^l] \) and
\[
u_n^u = \sum_{m=0}^{N} b_m^u \quad \text{and} \quad \nu_n^l = \sum_{m=0}^{N} b_m^l, \quad m = 0, 1, \ldots, M. \quad (16)
\]

**Proof.** Due to Property 1, it is easy to see that the coefficients in Eq. (14) satisfy
\[
B(s, b^0 + b_1^1) = \sum_{m=0}^{M} (u_n^u + u_n^l) s^m = \sum_{m=0}^{M} u_n^u s^m + \sum_{m=0}^{M} u_n^l s^m,
\]
which means that the parameters in Eq. (17) can be summed; thus, we have
\[
u_n^u = \sum_{m=0}^{N} b_m^u \quad \text{and} \quad \nu_n^l = \sum_{m=0}^{N} b_m^l, \quad m = 0, 1, \ldots, M. \quad \blacksquare
\]

Condition (2) of Theorem 3 transforms the 2-D polynomial
\[
\begin{align*}
B(s, b^0 + b_1^1) &= \sum_{m=0}^{M} (u_n^u + u_n^l) s^m = \sum_{m=0}^{M} u_n^u s^m + \sum_{m=0}^{M} u_n^l s^m, \\
&= b_0^u + b_1^u (j\omega) + b_2^u (j\omega)^2 + b_3^u (j\omega)^3 + b_4^u (j\omega)^4 + b_5^u (j\omega)^5 + \cdots,
\end{align*}
\]
and for \( n = 0, 1, \ldots, N \), where \( b_m^u \) and \( b_m^l \) are given in Eq. (1). According to Property 1, we can rearrange Eq. (20) according to real parts and imaginary parts, and we get Eq. (20).

Notice that \( u_n^u(\omega) \) and \( u_n^l(\omega) \) in Eq. (20) are monotonic functions of the frequency \( \omega \), so we have
\[
u_n^u(\omega) \leq u_n^u(\omega, b_1^0) \leq u_n^l(\omega), \quad \text{for any } \omega \quad \text{and} \quad b_1^0 \in [b_1^u, b_1^l], \quad i = 1, \ldots, M.
\]

The property is the same as that of \( v_n^u(\omega) \) and \( v_n^l(\omega) \) in Eq. (20). Thus, the polynomial in Eq. (15) is an interval polynomial with the boundaries of Eq. (19), and the stability condition (18) is necessary and sufficient for \( B(j\omega, z, b) \).

\]

Let
\[
b_0^u(j\omega) = b_0^u + b_1^u(j\omega) + b_2^u(j\omega)^2 + b_3^u(j\omega)^3 + b_4^u(j\omega)^4 + b_5^u(j\omega)^5 + \cdots,
\]
and
\[
b_0^l(j\omega) = b_0^l + b_1^l(j\omega) + b_2^l(j\omega)^2 + b_3^l(j\omega)^3 + b_4^l(j\omega)^4 + b_5^l(j\omega)^5 + \cdots.
\]

Now, the number of vertex polynomials is reduced to \( 4^{(N+1)} \) from \( 2^{(2M+1)(N+1)} \) by the theorem, and correspondingly, the number of edge polynomials is reduced to \( \left( \begin{array}{c} 4^{(N+1)} \\ 2 \end{array} \right) \). This means that to complete the stability test of a family of interval quasipolynomials, the total number of computations for vertex polynomials is \( 4^{(N+1)} \), and it is \( \left( \begin{array}{c} 4^{(N+1)} \\ 2 \end{array} \right) \) for edge polynomials. The authors in [3] did not obtain the results of the proposed Theorem 4.

Based on Theorem 4, a test algorithm for testing the robust stability of quasipolynomials can be obtained.

**Step 1:** Transform the interval quasipolynomial into a 2-D interval polynomial using Eq. (7).

**Step 2:** Construct the vertex polynomials by selecting the coefficients \( b_i^u(s) \) as follows:
\[
B_i(s, z) = \sum_{n=0}^{N} b_i^u(s) z^n, \quad i \in \{1, 2, 3, 4\},
\]
and
\[
b_i^u(s) \in \{b_0^u(s), b_2^u(s), b_4^u(s), b_6^u(s)\}, \quad (24)
\]
and \( b_i^u(s) \), \( b_i^u(s) \), \( b_i^l(s) \), and \( b_i^l(s) \) are the Kharitonov polynomials.
other polynomial sequences are computed by Eq. (26).

\[ b_n(s) = b_n^+ + b_n s + b_n s^2 + b_n s^3 + b_n s^4 + b_n s^5 + b_n s^6 + \ldots \]  

(25-1)

\[ b_n(s) = b_n^+ + b_n s + b_n s^2 + b_n s^3 + b_n s^4 + b_n s^5 + b_n s^6 + \ldots \]  

(25-2)

\[ b_n(s) = b_n^+ + b_n s + b_n s^2 + b_n^+ s^3 + b_n s^4 + b_n s^5 + b_n s^6 + \ldots \]  

(25-3)

\[ b_n(s) = b_n^+ + b_n s + b_n^+ s^2 + b_n^+ s^3 + b_n^+ s^4 + b_n s^5 + b_n^+ s^6 + \ldots \]  

(25-4)

for \( n = 0, 1, \ldots, N \), where \( b_n^+ \) and \( b_n^- \) are given by Eq. (1).

Step 3: Construct edge polynomials by considering all of the different combinations of vertex polynomials using Eq. (23).

Step 4: Test the stability of the obtained vertex polynomials.

Step 5: Test the stability of the obtained edge polynomials.

If the stability conditions in Step 4 and Step 5 are satisfied, the given interval quasipolynomial is stable.

The stability test of vertex polynomials and edge polynomials is provided in the following section.

IV. STABILITY TEST FOR VERTEX AND EDGE POLYNOMIALS

Extending the results of [15] to the case of Hurwitz-Schur stability, we obtain the following theorem, which tests the Hurwitz-Schur stability of the extreme polynomials of the interval polynomial \( B(s, z, b) \). The polynomial sequence in Table 1 is constructed by

\[ b_k(s) = \begin{bmatrix}
  b_{i,0}(s) & b_{i,-1,N+i+1}(s) \\
  b_{i,1,N+i+1}(s) & b_{i,-1,k}(s)
\end{bmatrix} \]  

(26)

for \( i = 1, \ldots, N-1 \); \( k = 0, 1, \ldots, N-i \), where \( b_{i,0}(s) = b_{i,0}^+(s) \), and \( b_{i,k}^-(s) \) is defined by Eq. (23).

Theorem 5. (Extreme Test) Construct a complex Schur table as shown in Table 1. The first two rows of the table list the coefficient polynomials of \( B_i(s, z) \) in Eq. (23), and other polynomial sequences are computed by Eq. (26).

Then the extreme polynomial \( B_j(s, z) \) is stable if and only if for \( i = 1, 2, 3, \ldots, N-1 \)

(a) \( b_{i,0}(s) \pm b_{i,-1,N-i}(s) \neq 0 \), \( \Re s \geq 0 \), \hspace{1cm} (27a)

(b) \( |b_{i,0}(j\omega)| > |b_{i,-1,N-i}(j\omega)| \), for all \( \omega \in R \). \hspace{1cm} (27b)

Though condition (b) of Theorem 5 requires testing of all the frequencies \( \omega \in R \) in theory, in fact, for a practical system, we only ensure that the system is stable in some given frequency range, instead of the whole real space \( R \).

The proof of Theorem 5 can be found in [8,9].

The Hurwitz-Schur stability of the edges of an interval 2-D polynomial can be tested by means of the following theorem. For the two vertex polynomials \( B_i(s, z) \) and \( B_j(s, z) \) of \( E_{ij}(s, z) \), the interval 2-D polynomial, we define the Complex Schur Table Test in the following Theorem.

Theorem 6. (Edge Test) Construct a complex Schur table as in Table 2; the first two rows of the table show the coefficient polynomials of the two stable vertex polynomials \( B_i(s, z) \) and \( B_j(s, z) \). Let

\[ b_{i,k}(s) = \begin{bmatrix}
  b_{i,0,0}(s) & b_{i,0,N-1-k}(s) \\
  b_{i,1,N-1-k}(s) & b_{i,1,k}(s)
\end{bmatrix} \]  

for \( k = 0, 1, \ldots, N-1 \); \hspace{1cm} (28)

the other polynomial sequence in Table 1 can be constructed by

\[ b_{i,k}(s) = \begin{bmatrix}
  b_{i,-1,0}(s) & b_{i,-1,N+i+1-k}(s) \\
  b_{i,1,N+i+1-k}(s) & b_{i,1,k}(s)
\end{bmatrix} \]  

(29)

for \( i = 2, \ldots, N-1 \); \( k = 0, 1, \ldots, N-1 \). Let \( s = j\omega \); then, the edge polynomial \( E_{ij}(s, z) \) is stable if and only if for \( i = 1, 2, 3, \ldots, N-1 \),

\( |b_{i,0}(j\omega)| > |b_{i,-1,N-i}(j\omega)| \), for all \( \omega \in R \). \hspace{1cm} (30)

The proof of the theorem can be found in [8,9].

Remark. The main differences between the vertex test and edge test are the computation of the first two polynomials’ sequences in the test table and the test conditions in (27) and (30).

Table 1. Vertex test table for \( B_i(s, z) \).

<table>
<thead>
<tr>
<th>Row(i)</th>
<th>( z^0 )</th>
<th>( z^1 )</th>
<th>( z^{N-1} )</th>
<th>( z^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( b_{0,0}(s) )</td>
<td>( b_{0,1}(s) )</td>
<td>( b_{0,N-1}(s) )</td>
<td>( b_{0,N}(s) )</td>
</tr>
<tr>
<td>0</td>
<td>( b_{1,0}(s) )</td>
<td>( b_{1,1}(s) )</td>
<td>( b_{1,N-1}(s) )</td>
<td>( b_{1,N}(s) )</td>
</tr>
<tr>
<td>1</td>
<td>( b_{2,0}(s) )</td>
<td>( b_{2,1}(s) )</td>
<td>( b_{2,N-1}(s) )</td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td>( b_{N-1,0}(s) )</td>
<td>( b_{N-1,1}(s) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| N-1    | \( b_{N,0}(s) \) | \( b_{N,1}(s) \) |

Table 2. Edge test table for \( E_{ij}(s, z) \).

<table>
<thead>
<tr>
<th>Row(i)</th>
<th>( z^0 )</th>
<th>( z^1 )</th>
<th>( z^{N-1} )</th>
<th>( z^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( b_{0,0}(s) )</td>
<td>( b_{0,1}(s) )</td>
<td>( b_{0,N-1}(s) )</td>
<td>( b_{0,N}(s) )</td>
</tr>
<tr>
<td>0</td>
<td>( b_{1,0}(s) )</td>
<td>( b_{1,1}(s) )</td>
<td>( b_{1,N-1}(s) )</td>
<td>( b_{1,N}(s) )</td>
</tr>
<tr>
<td>1</td>
<td>( b_{2,0}(s) )</td>
<td>( b_{2,1}(s) )</td>
<td>( b_{2,N-1}(s) )</td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td>( b_{N,0}(s) )</td>
<td>( b_{N,1}(s) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| N-1    | \( b_{N,0}(s) \) | \( b_{N,1}(s) \) |
The above theorems are sufficient conditions for the edges of interval quasipolynomials as in Eq. (5), according to Theorem 1 and Theorem 2. Limited by the paper size, the example for the application of the proposed theorems could not be provided.

VI. CONCLUSIONS

In this brief paper, sufficient conditions for the stability of interval quasipolynomials have been established, based on the stability of 2-D \( s \)-\( z \) interval polynomials. The stability test for interval quasipolynomials is performed in the 2-D \( s \)-\( z \) domain instead of the classical \( s \) domain. A method for constructing 2-D hybrid interval polynomials by using interval quasipolynomials has been provided. A test set for vertex polynomials and edge polynomials has been developed by extending the Kharitonov extreme test idea to complex coefficient polynomials, and the number of vertex polynomials and edge polynomials has been greatly reduced to \( 4^{(N+1)} \) from \( 2^{(M+1)(N+1)} \) and to \( \binom{4^{(N+1)}}{2} \) from \( \binom{2^{(M+1)(N+1)}}{2} \), respectively. Extreme test and edge test algorithms for testing the robust stability of 2-D hybrid interval polynomials have also been provided. In further research on this topic, we will study an algebraic algorithm for testing the robust stability of quasipolynomials with uncertain delays.

REFERENCES